



# IALA GUIDELINE

1035

## AVAILABILITY AND RELIABILITY OF AIDS TO NAVIGATION - THEORY AND EXAMPLES

**Edition 2.0**

**December 2004**



# DOCUMENT REVISION

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Revisions to this IALA Document are to be noted in the table prior to the issue of a revised document.

Date	Page / Section Revised	Requirement for Revision
December 2004	Full Review and update	Maintenance of document



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# 1 INTRODUCTION

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## 1.1 PURPOSE AND SCOPE

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Availability is a key performance indicator which together with others can be used as a management tool that can be used to measure, analyse and monitor the performance of aids to navigation and/or specific systems and equipment. The information obtained can be used to:

- show accountability to government and stake holders;
- demonstrate the efficiency and effectiveness of the service being provided;
- compare the performance of:
  - similar systems or equipment in different locations;
  - contract and internally provided services (where both are engaged in substantially similar work).
- amend:
  - system designs;
  - procurement decisions;
  - equipment choices;
  - maintenance procedures and practices.
- increase or reduce maintenance effort;
- extend maintenance intervals.

The Guideline shows a method of calculating these performance indicators, with a view to enabling IALA Members to provide a cost effective AtoN service. The guidelines may be used by:

- service providers to calculate actual AtoN availability and reliability;
- system designers to define expected system availability and reliability and any requirement for redundancy to ensure that the availability objectives set by management can be met;
- maintenance managers to define measurable performance targets for systems and sub systems to ensure that the objectives set by management can be met.

The outlined methodology may be used to calculate the predicted reliability of a single aid to navigation consisting of several statistically independent subsystems each with its own level of reliability expressed as Mean Time Between Failure (MTBF).

The methodology may also be used to calculate the predicted reliability of a system of aids, consisting of a number of individual aids.

## 1.2 ECONOMIC ASPECTS OF RELIABILITY AND AVAILABILITY

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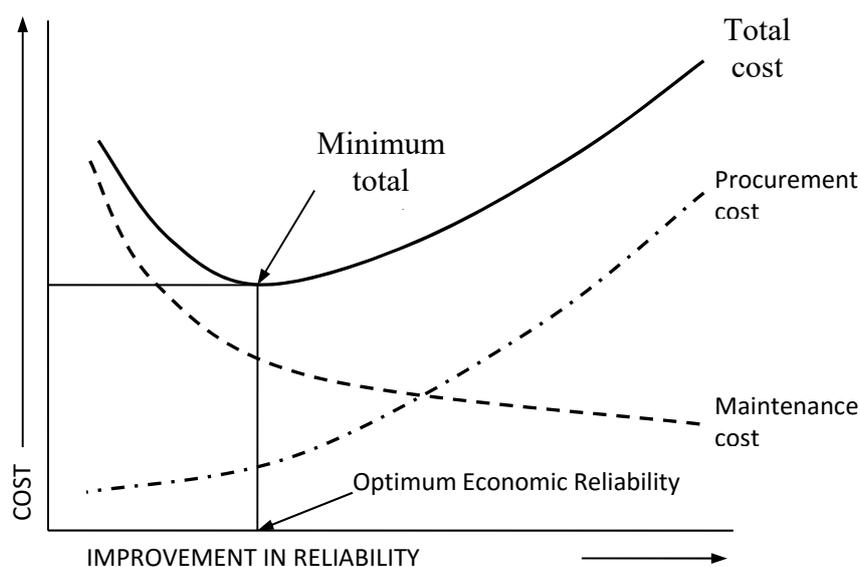
An AtoN availability objective, determined on the basis of the operational role of the aid, can be reached by an appropriate combination of maintenance, logistics and equipment reliability.

The fact that reliability and lighthouses have become synonymous can, to a great extent, be attributed to the relative simplicity of the aids and the availability of a supply of reasonably priced labour to operate and maintain them. As technology advances, better aids and services become available. These aids and services are usually more complex from an engineering standpoint, resulting in more dependence on equipment reliability in lieu of human reliability. In recent years, rapidly increasing labour costs associated with operating lighthouses and

lightships have resulted in the automation of many of these aids. As a result, aids continue to become more dependent on equipment reliability.

In general, reliability costs money, and the cost of equipment procurement – including development design and manufacture – increases with increasing reliability. The latter can be achieved by raising the quality of the whole design and manufacturing process, and also, as is common practice with Lighthouse Authorities, by preventive maintenance and providing the system with redundancy in the form of one or more standby equipment's in reserve which can be brought into service on failure of the working equipment, or in the form of active redundancy, wherein all means for performing a given function are operating simultaneously. This will also increase initial capital costs.

However, unreliability also carries a cost penalty in terms of increased maintenance costs, spares provisions, and, where appropriate, loss of revenue or other related costs arising from failure. This relationship is complex, but as a general principle, there is a trade-off situation where the cost of reliability and the cost of failure are minimised



**Figure 1** *Procurement and maintenance costs vs improvement in reliability*

This is illustrated by the curves of Figure 1, which shows the procurement costs increasing with reliability and the corresponding falling costs associated with maintenance. These combine to give a curve of total, or whole life costs –sometimes called ‘Cost of ownership’ – which has an optimum minimum value at a certain level of reliability. This minimum cost is not necessarily the governing factor in determining the degree of reliability required; there are other factors, such as safety, which may require a higher reliability regardless of the increased costs.

High standards of aid reliability/availability may initially be expensive, but can be economical when considering the life time costs. Therefore, all factors relating to the subject should be considered by Lighthouse Authorities. In some instances, it may be necessary to abolish some AtoN in order to concentrate available resources on a reduced number of aids providing an acceptable level of service.

### 1.3 PERFORMANCE INDICATORS

The following performance indicators are considered to be applicable to AtoN systems:

#### 1 Availability

This is the probability that an aid to navigation or a system of aids to navigation as defined by the Competent Authority is performing its specified function at any randomly chosen time. This is expressed as a percentage



of total time that an aid to navigation or a system of aids to navigation should be performing their specified function. This is a measure of the service provided to the mariner.

## 2 Reliability

This is the probability that an aid to navigation, when it is available, performs a specified function without failure under given conditions for a specified time. This is a measure of the performance of AtoN equipment.

## 3 Mean Time Between Failures (MTBF)

This is the average time between successive failures of a repairable AtoN, system or part of a system. It is a measure of reliability.

## 4 Mean Time to Repair (MTTR)

This is the time it takes to restore an aid to normal operation after it fails. This is a measure of an Authority's administrative arrangements, resources and technical capability to rectify a fault. Effectively this is a measure of the performance of the repair team.

## 5 Continuity

This is the probability that an aid to navigation or system will perform its specified function without interruption during a specified time. This is mainly used for radionavigation systems.

## 2 AVAILABILITY

### 2.1 INTRODUCTION

Use of the 'AVAILABILITY' parameter (A) is a good method for defining the level of service a mariner can expect from an aid to navigation. Furthermore, the numerical value of availability can be used to determine, in precise quantifiable terms, the sum total of all the relevant characteristics of the design, engineering, procurement and quality assurance procedures involved in the provision of aids to navigation, together with the necessary logistics and maintenance.

It should be emphasised that, within the framework of what follows, availability concerns only the ability of an aid to operate as advertised in nautical documents and does not consider external factors such as reduced meteorological visibility. However, it is true to say that at the design stage the intensity of a light will be chosen according to visibility conditions prevailing locally.

It is also true to say that a light of say 1000 candelas (cd) operating as advertised will have the same luminous output irrespective of the visibility and that it is the duty of the mariner to adjust his expectations and behaviour according to the weather conditions.

Navigation lights should be categorised in accordance with IALA Recommendation O-130 on Categorization and Availability Objectives for Short Range Aids to Navigation.

### 2.2 AVAILABILITY TARGETS

Recommended targets for AtoN availability are provided in IALA Recommendation O-130 on Categorization and Availability Objectives for Short Range Aids to Navigation.

It should be noted that these long term availability objectives are not appropriate for presentation in nautical publications as they cannot represent a commitment of the Lighthouse Authorities toward seafarers in any particular short term period.

### 2.3 AVAILABILITY CALCULATION

The parameter A may be calculated by dividing the total time during which the aid has been operating correctly (i.e. Total Time – Down Time), by the Total Time during which the Aid should have performed correctly. All times are expressed in hours and are measured over the same time period.



The availability (A) may be calculated by dividing the total time during which the aid has been operating correctly (i.e. Total Time – Down Time), by the Total Time during which the Aid should have performed correctly. All times are expressed in hours and are measured over the same time period.

$$A = \frac{\text{Total Time} - \text{Down Time}}{\text{Total Time}} \text{ (hours)}$$

Equation 1    Availability (1)

This can be expressed in a number of ways, as follows:

$$A = \frac{\text{Up Time}}{\text{Total Time}}$$

Equation 2    Availability (2)

$$A = \frac{\text{Service Time} - \text{Out of Service Time}}{\text{Service Time}}$$

Equation 3    Availability (3)

The Internationally recognised formula for calculating availability is:

$$A = \frac{MTBF}{MTBF + MTTR}$$

Equation 4    Availability (International)

Total Time for an Aid to Navigation over x years is calculated as:

$$\text{Total Time} = (365 \times 24 \times x) \text{ hours}$$

Equation 5    Total Time

Down Time should be recorded in accordance with the 'IALA Recommendation O-130 on Categorization and Availability Objectives for Short Range Aids to Navigation.

As an example: It should be noted that for lights Down Time as per Total Time in the formula does not differentiate between periods of daylight and darkness.

Percentage availability is also used. It is defined as 100 times the availability:

$$A_{\%} = 100A$$

Equation 6    Percentage availability

In accordance with IALA Recommendation O-130 on Categorization and Availability Objectives for Short Range Aids to Navigation, Availability Objectives are calculated over a three-year continuous period, unless otherwise specified.

### 3 RELIABILITY OF SYSTEMS

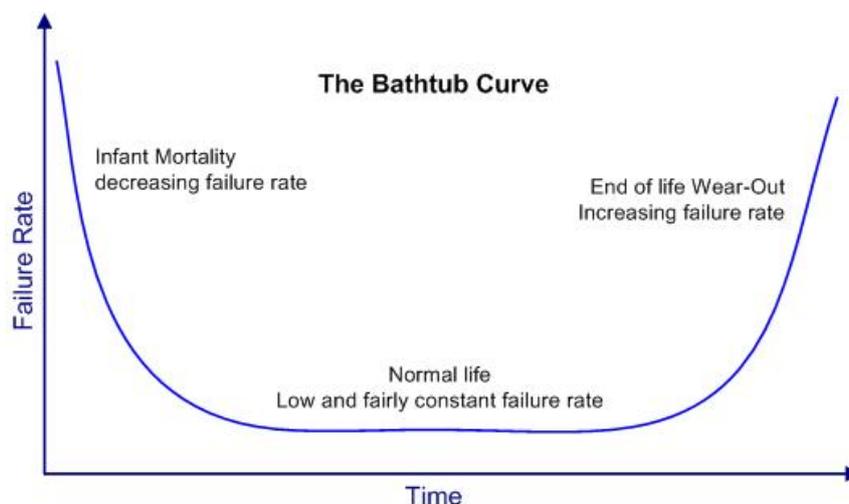
#### 3.1 INTRODUCTION

Systems are made up of subsystems, and subsystems are made up of components which all have their own individual reliability. Therefore, the total system reliability is directly dependent upon the reliability of each component within the system as well as how these are put together to form subsystems.

The term reliability means the probability that a system, subsystem, or a component, will properly perform its desired function(s) over a given time period.

The failure rate, also known as the hazard rate, being the number of failures of an item per unit time where the item has been performing its required function(s), is a useful reliability indicator.

In real life, the failure rate is not constant over the whole lifetime of a system. This is illustrated in Figure 2.



**Figure 2** *Failure rate changes over the lifetime of a population of items*

This curve is known as the bathtub curve. The curve does not depict the failure rate of a single item or system, but describes the relative failure rate of an entire population of items or systems over time.

Some individual items will fail relatively early (infant mortality failures), others will last until wear-out, and some will fail during the relatively long period typically referred to as normal life.

Failures during infant mortality are typically caused by material defects, errors in assembly, design errors, defective component etc.

Normal life failures are normally considered to be random cases of ‘stress exceeding strength’. In some cases, a number of failures considered as normal life failures during the early stages of normal life are actually infant mortality failures.

Wear-out is a fact of life due to fatigue of materials such as lack of lubrication in bearings, filament lamp reaching the end of service life etc.

Thus the bathtub curve illustrates the three key periods, or failure modes, in the lifetime of a population of items.

It is very important that managers understand how the failure mode and the failure rate of a population changes with time and how collected data on equipment failures should be interpreted to give a better understanding of the behaviour of the population.

### 3.2 MEAN TIME BETWEEN FAILURES

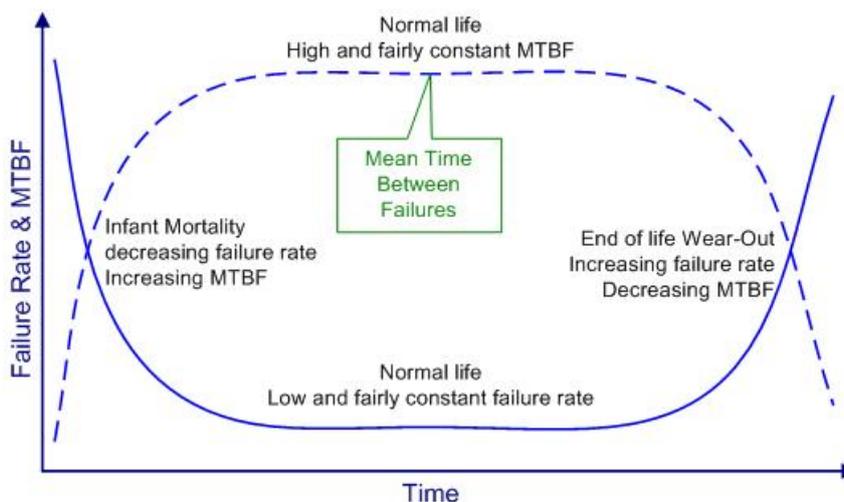
The term Mean Time Between Failures (MTBF) is often used (and misused) to give a description of the reliability of products and systems. MTBF may be calculated from data on observed failures and is basically defined as:

$$MTBF = \frac{1}{Failure\ Rate}$$

Equation 7 *Mean Time Between Failure (1)*

So if you have a population of 100 units and experience 50 failures within one year, the Mean Time Between Failures in that population is 2 years. MTBF of a larger system or product may be predicted during the design phase based on data on the MTBF of each of its subsystems and/or components.

However MTBF and its definition in Equation 7 is only applicable when the underlying failure distribution function has a constant failure rate (e.g. exponential distribution of failures) which is not the case during both infant and wear-out parts of the life cycle. This can be illustrated by depicting MTBF together with the bathtub curve:



**Figure 3** *MTBF is only constant during the so-called normal part of the life cycle*

As can be seen from Figure 3, it only makes sense to use MTBF as a measurement parameter and basis for planning while the population is in the ‘normal life’ part of its life cycle and where the failure rate is constant.

### 3.3 DEALING WITH INFANT MORTALITY FAILURES (DESIGN IMPROVEMENTS)

It is up to manufacturers and system designers to take appropriate measures in the form of proper design efforts and testing activities prior to delivering equipment for service. This should include burn-in testing and root-cause analysis of any failures encountered followed by proper feedback to the designers and or manufacturers.

During procurement, AtoN authorities should ensure that their required minimum level of testing is written into procurement specifications.

The AtoN manager should be conscious of the fact that in the early life of a population of items, infant mortality failures will occur, and data on these failures should be identified separately from the failures detected at later stages of the life cycle in order to be able to calculate the true ‘normal life’ MTBF.

### 3.4 DEALING WITH WEAR-OUT FAILURES (PREVENTIVE MAINTENANCE)

As mentioned, end of life wear-out failures mainly occur due to fatigue of materials. One example of this type of failures is the unavoidable ultimate failure of an incandescent lamp after a successful Service Life. This type of failures can obviously in many cases be prevented through preventive maintenance, replacing items that potentially wear out prior to their actual failure.

The importance of proper preventive maintenance is obvious, however a proper balance between resources used for planned maintenance and the resulting improvement in reliability must be ensured.

Therefore, careful planning of the preventive maintenance, based on a deep understanding of the wear-out mechanisms taking place within the population at hand, must be ensured.

Manufacturers should state the so-called Service Life of their products, defining and describing what wear-out mechanisms are dominant in their products and how long the product can be expected to work under the specified operating conditions before failing due to wear-out of some kind.

Note that the Service Life parameter is not the same as MTBF.

### 3.5 DEALING WITH NORMAL LIFE FAILURES (REPAIR TEAM)

As mentioned before, normal life failures mainly occur due to random cases of ‘stress exceeding strength’ e.g. when a filament lamp fails due to a power supply transient.

Due to the random nature of this type of failures, preventive maintenance is useless for preventing such failures.



The only way of reducing the number of these random failures is to increase the reliability of the design at hand by adjusting the design. This could take the form of optimising the mechanical and electrical strength of some parts of the design or increase the cooling of another part to prevent overheating etc.

Given the level of reliability of a given item or subsystem within a larger system, one can improve the reliability of the system as a whole by introducing functional redundancy at the sub system level. This issue will be treated further later in this document.

When a given system fails to operate due to a random normal life failure, it is important to have the necessary repair resources in place for ensuring timely recovery, so that the desired system availability can be achieved.

### 3.6 SYSTEM RELIABILITY CALCULATION

MTBF may be calculated by dividing Total Time minus Down Time by the number of failures, where MTBF and all times are expressed in hours (see Equation 1  $A = \frac{\text{Total Time} - \text{Down Time}}{\text{Total Time}}$  (hours)).

The availability of an aid to navigation may also be calculated by dividing the MTBF by the sum of the MTBF and the Mean Time to Repair (MTTR).

If Equation 3 ( $A = \frac{MTBF}{MTBF + MTTR}$ ) is rewritten to obtain MTBF:

$$MTBF = MTTR \left( \frac{A}{1 - A} \right)$$

*Equation 8 Mean Time Between Failure (2)*

which is useful for system designers to calculate the required reliability (MTBF) of systems and subsystems to ensure that the availability objectives set by management can be obtained with a given MTTR.

### 3.7 MEAN TIME TO REPAIR (MTTR)

MTTR can be calculated from the down time, expressed in hours, divided by the number of failures.

$$MTTR = \left( \frac{\text{Down Time}}{\text{Number of Failures}} \right)$$

*Equation 9 Mean Time to Repair (1)*

MTTR can be lowered by use of modular systems that permit fast repair by replacement of defective units and by use of devices that ease service technicians' understanding and facilitate fast identification of defective components.

### 3.8 RELIABILITY IMPROVEMENT

The reliability of an aid to navigation system can be improved by using good quality system equipment components and by choosing a suitable aid system operating philosophy.

Equipment related improvement approaches affect both the MTBF and the MTTR portions of the availability equation. Specification of components with high reliability and the judicious application of redundancy of those components that are least reliable improves system performance and overall aid reliability. An alternative approach is to simplify the system by using devices with fewer parts that are simple to understand and replace, leaving less to fail, resulting in higher aid MTBF.

The response of Maintenance to AtoN failures, affects the MTTR portion of the Availability equation. Immediate dispatch of repair teams and use of fast transport such as helicopters or high-speed boats to deliver repair teams can reduce what is most often the dominant portion of MTTR, the transit time to the aid. Electronic monitoring of aids may be used to detect trends which could lead to failure, and enable planned maintenance to pre-empt failures and improve the effectiveness of repair crews.



Decisions about system operation can compensate somewhat for low MTBF performance in the shortened interval. Provision of spare parts on-site for repairable items and operating a rigorous management information system to maintain spare parts integrity on-site ensures repairs can be made with one trip to the aid.

Methods selected for system operation also help determine the service infrastructure necessary to operate and maintain the aid system. Distribution of resources (tenders, vehicles, aircraft, people and workshops) and failure response policy has direct bearing on MTTR.

An established preventative maintenance schedule for each aid, and an established training programme for service technicians and managers is vital to achieve the hardware system's highest potential MTBF. When setting operational requirements, a Lighthouse Authority should simplify the performance requirements as much as practical; while highly complex power, control, and signal systems can be provided by technical experts, they may not be desirable in terms of cost, levels of maintenance, and technician skill needed for reliable operation.

Section 4 presents methods of calculation of the predicted reliability of equipment based on the reliability of its components. By comparing the expected MTBF objective derived from such an analysis to even a fairly rough estimate of the MTBF of existing aids, the MTBF prediction calculation can be verified. A decision is then often possible on whether an existing equipment satisfies the objective, whether any standby element(s) should be removed or added, whether more reliable equipment should be sought and to what extent it would not be more advantageous to introduce improved maintenance procedures.

IALA Recommendation E-105 on the need to follow national and international standards recommends that:

- Lighthouse Authorities make as much use as possible of available national and international standards, which can provide a very acceptable level of quality in their equipment procurement and specifications.  
Preference should also be given to suppliers who have some form of official certification approval of their quality assurance procedures from their National Standards Authority.
- Lighthouse Authorities are further recommended to consider the feasibility of setting up their own internal quality assurance procedures.

Bearing in mind the motto that you cannot manage what you cannot measure, attention must be paid to gathering reliability data. This provides a measurable basis for assessing the performance of AtoN. IALA Guideline 1037 on Data Collection for Aids to Navigation Performance Calculation provides further information on performance data collection and analysis.

### 3.9 RISK ANALYSIS

Risk analysis recognises two components – the probability of failure and the consequences of failure – and the level of risk is the product of these two factors. Clearly different considerations apply to relatively frequent minor incident of little consequence and a single rare incident with catastrophic consequences.

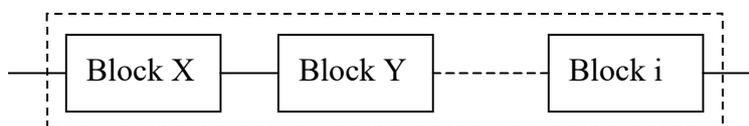
Several different aids to navigation configurations in a given waterway may provide the same effective coverage. Solution 'A' may rely on a relatively few, expensive, highly reliable aids, while Solution 'B' may consist of numerous, less costly, less reliable aids. Both solutions may be equally effective in providing the mariner with the necessary navigation information. Solution 'C' may use the same aids as Solution 'A', but with a lower individual aid reliability due to stringent regulations governing the training and use of pilots. Experienced pilots generally have the capability of relying less on provided aids to navigation than a navigator without local knowledge.

Considerable problems have been found to arise in the application of risk analysis to the aids to navigation provided by Lighthouse Authorities because of the complex and largely indeterminate variables involved. However, there is a software program 'IALA Waterways Risk Analysis Program' (IWRAP), available from IALA, to assist in this process.

## 4 SYSTEM RELIABILITY MODELS

### 4.1 MODELLING SYSTEM RELIABILITY

A larger system consisting of a number of subsystems may be modelled during the normal life / constant failure rate section of the bath-tub curve by a number of individual functionally interconnected 'blocks' as shown in Figure 4.



**Figure 4** *Individual functionally interconnected 'blocks'*

Modelling system reliability using individual and statistically independent subsystems.

During normal life, each block may be assumed to have its own level of reliability, given that the blocks are statistically independent of each other.

The statistical distribution of random normal life failures is traditionally modelled by the exponential distribution function given by:

$$f(t) = \left( \frac{1}{MTBF} \right) e^{-(1/MTBF)t}$$

**Equation 10** *The exponential distribution function*

The reliability function of this distribution function is given by:

$$R(t) = e^{-(1/MTBF)t}$$

**Equation 11** *The reliability function*

The reliability function can be used to calculate the probability that a block is operating properly at any given point in time since it started operating.

The exponential distribution function is widely used due to its simplicity, and not necessarily because it is an accurate model of all types of random failures, which it is not. Other distribution functions such as the Weibull distribution are more versatile and widely used today, but require typically dedicated software packages to be put to use due to their complexity.

It is important to understand that the reliability analysis is based upon an exponential distribution function of failures.

When performing reliability analysis as described later in this document, it is important to understand and keep in mind the following basic assumptions and constraints:

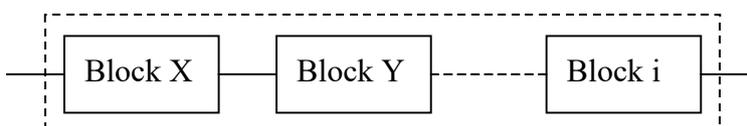
- all systems may be modelled by a number of subsystems (blocks) connected in series or parallel or as a combination of both;
- the various blocks are statistically independent;
- each block is operating in 'normal life' and has a constant MTBF;
- for a given block the MTTR is far shorter than its MTBF;
- the difference between MTBF and MTTR is negligible since MTTR is far shorter than MTBF;
- the probability that a switching device operates correctly is assumed to be constant and equal to 1;

- a repaired item is equal to a new item in terms of MTBF;
- an item put into service from time to time (e.g. a standby element) has the same MTBF as one operating continuously.

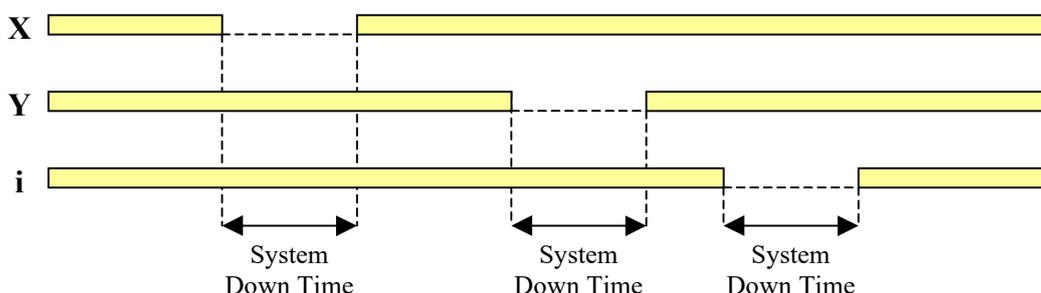
In the remainder of this section practical methods of calculating the MTBF of larger systems based on the MTBF of its individual subsystems (blocks) is shown.

## 4.2 BLOCKS IN SERIES

The system operates if, and only if, each of the blocks is in good running order.



**Figure 5** *Blocks in series*



**Figure 6** *Presentation of operating time*

The MTBF of the system ( $MTBF_{SYS}$ ) is then given by the following formula:

$$\frac{1}{MTBF_{SYS}} = \frac{1}{MTBF_X} + \frac{1}{MTBF_Y} + \dots + \frac{1}{MTBF_i}$$

**Equation 12** *System MTBF*

Where

$MTBF_i$  is the Mean Time Between Failures of the  $i$ 'th block.

The MTTR of the system ( $MTTR_{SYS}$ ) is given by the following formula:

$$MTTR_{SYS} = MTBF_{SYS} \sum_{n=1}^i \frac{MTTR_n}{MTBF_n} \quad (\text{Refer to section A 1})$$

**Equation 13** *System MTTR*

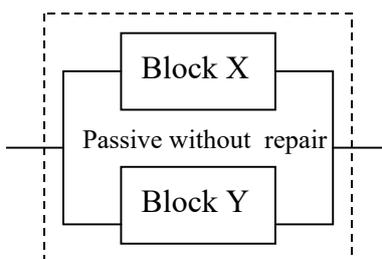
Where:

$p$  is the probability that Y doesn't start operating correctly when X fails.

## 4.3 BLOCKS IN PARALLEL, PASSIVE REDUNDANCY

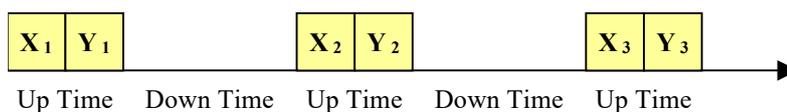
### 4.3.1 PASSIVE REDUNDANCY. WITHOUT REPAIR

Block Y is put into service only if block X fails, and block X is repaired only after failure of block Y.



**Figure 7** *Passive redundancy - without repair*

The succession of up times and down times is as follows:



**Figure 8** *Presentation of passive redundancy - without repair*

$$MTBF_{SYS} = MTBF_X + (1 - p)MTBF_Y \quad (\text{Refer to section A 2})$$

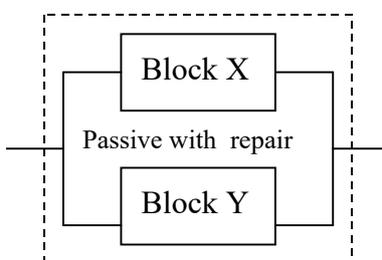
**Equation 14** *MTBF of passive system without repair*

where:

p is the probability that Y doesn't start operating correctly when X fails

#### 4.3.2 PASSIVE REDUNDANCY. WITH REPAIR

The block Y is put into service only if the block X fails and operates only while X is under repair.



**Figure 9** *Passive redundancy - with repair*

The MTBF of the system ( $MTBF_{SYS}$ ) is then given by the following formula:

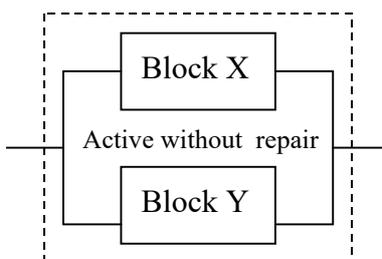
$$MTBF_{SYS} = \frac{MTBF_X + MTTR_X}{(p + (1-p)) \frac{MTTR_X}{MTBF_Y}} \quad (\text{Refer to section A 3})$$

**Equation 15** *MTBF of passive system with repair*

## 4.4 BLOCKS IN PARALLEL, ACTIVE REDUNDANCY

### 4.4.1 ACTIVE REDUNDANCY WITHOUT REPAIR WHEN ONLY ONE BLOCK HAS FAILED

X and Y run at the same time when X or Y alone is enough to produce the function required from S (a failure of the system occurs only when X and Y are out of order).



**Figure 10** *Active redundancy - without repair when only one block has failed*

If X and Y are identical then, if the time between failure was fixed (i.e. failures don't occur at random),  $MTBF_{SYS}$  would be equal to  $MTBF_X$  and the redundancy would not carry advantages.

Active redundancy will prove beneficial if the dispersion of the running times is of some importance.

If X and Y are identical with a constant failure rate  $MTBF_{SYS}$  is given by the following formula:

$$MTBF_{SYS} = \frac{3-2p}{2} MTBF_X \quad (\text{Refer to section A 4})$$

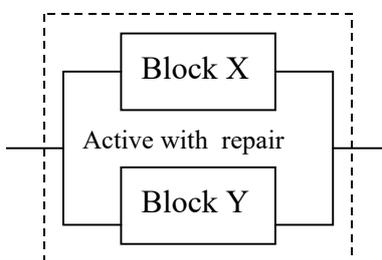
**Equation 16** *MTBF of a system where blocks X & Y are identical with a constant failure rate*

Where:

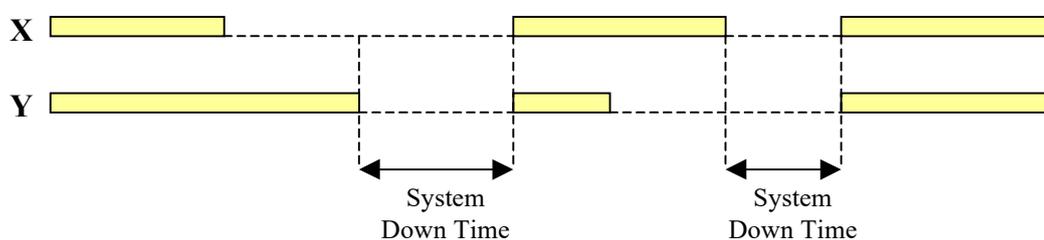
p is the probability that the second block becomes ineffective as soon as a failure occurs on one block.

#### 4.4.2 ACTIVE REDUNDANCY WITH REPAIR OF A FAILED ITEM

X and Y operate simultaneously when X or Y alone would be enough to ensure that the function required from the system S is performed.



**Figure 11** *Active redundancy - with repair when only one block has failed*



**Figure 12** *Presentation of active redundancy - with repair*

$MTBF_{SYS}$  is given by the following formula:



$$MTBF_{SYS} = \frac{1}{\left(\frac{1}{MTTR_X} + \frac{1}{MTTR_Y}\right)} \left[ \frac{1-IS}{IS} \right] \quad (\text{Refer to section A 5})$$

Equation 17 *Active redundancy with repair when only one block has failed*

where:

$$IS = \frac{MTTR_X}{(MTBF_X + MTTR_X)} \times \frac{MTTR_Y}{(MTBF_Y + MTTR_Y)}$$

## 5 EXAMPLES

### 5.1 METHODOLOGY TO ASSESS FAILURE RESPONSE TIME (FRT) IMPACT ON SERVICE AVAILABILITY

#### 5.1.1 COMPUTATION OF FAILURE RESPONSE TIME

Failure Response Time (FRT) can be derived from the availability (A) relationship between MTBF and MTTR:

(see Equation 3 ( $A = \frac{MTBF}{MTBF+MTTR}$ )), which gives Equation 8 ( $MTBF = MTTR \left( \frac{A}{1-A} \right)$ )

Solving for MTTR yields:

$$MTTR = \frac{MTBF(1 - A)}{A}$$

Equation 18 *Mean Time to Repair (2)*

Taking as an example the following case where values are given:

A	= 0.998
MTBF	= 14 000 hours
Mean Time To Report ( <i>MTT Report</i> )	= 5 hours
Mean Time To Prepare ( <i>MTT Prepare</i> )	= 4 hours
Mean Time To Transport ( <i>MTT Transport</i> )	= 11.5 hours
(Includes possible loss of time due to bad weather)	
Mean Time To Repair on Site ( <i>MTT Repair on Site</i> )	= 2 hours

Recognising that:

$$MTTR = \text{Mean Failure Response Time (MFRT)} + MTT \text{ Report} + MTT \text{ Prepare} + MTT \text{ Transport} + MTT \text{ repair on site}$$

Equation 19 *Composition of MTTR*

Inserting the assumed values yields:

$$MTTR = 5 + 4 + 11.5 + 2 + MFRT$$

$$MTTR = 22.5 + MFRT$$

Substituting this value for MTTR in Equation 18 yields:

$$22.5 + MFRT = \frac{14000(1-0.998)}{0.988}; 22.5 + MFRT = 28.1; MFRT = 28.1 - 22.5$$

$$MFRT = 5.6 \text{ hours.}$$



## 5.1.2 CONCLUSION

For the given values of MTBF and repair times, a Mean Failure Response Time of 5.6 hours selected by management would permit the lighthouse service to maintain the stated level of service ( $A = 0.998$ ) to the mariner.

If the computed value for MFRT is negative, it means that the MTBF is too low or the total MTTR is too high to provide the stated availability. The MTBF situation can be helped by specifying higher quality equipment, increasing equipment redundancy, or increasing preventive maintenance. The MTTR situation can be helped by reducing the various time components in the MTTR total.

It is possible that a certain aid to navigation does not seem to be very important and as a result may have been assigned a low availability and a large MFRT. Due to a failure of this aid, a vessel may be misled, run aground and be heavily damaged and severely injure the environment. The resulting cost could be enormous. A Lighthouse service should keep this in mind when setting an availability objective.

## 5.2 EXAMPLES ILLUSTRATING SELECTIVE REPAIR POLICIES

The following figures are typical MTBF times for AtoN components:

Power supply:	3 fault/year >4 seconds = 3000 hours
Flasher:	80,000 hours
Diesel gen.:	10,000 hours when prescribed maintenance is performed
Optic drive:	20 year approximately 200,000 hours
Lamp:	2000 – 4000 hours

### 5.2.1 BLOCKS IN SERIES

Power supply – Flasher - Lamp

$$\text{From Equation 12 } \left( \frac{1}{MTBF_{SYS}} = \frac{1}{MTBF_X} + \frac{1}{MTBF_Y} + \dots + \frac{1}{MTBF_i} \right)$$

$$\frac{1}{MTBF_{SYS}} = \frac{1}{3000} + \frac{1}{80000} + \frac{1}{2000} \Rightarrow MTBF_{SYS} = 1182 \text{ hours}$$

### 5.2.2 PASSIVE REDUNDANCY WITHOUT REPAIR

Diesel generators

Other examples:

- flashers as backup
- power supply with stand by diesel generator
- main light with a stand by light
- lampchanger
- twin filament lamp

From Equation 14 ( $MTBF_{SYS} = MTBF_X + (1 - p)MTBF_Y$ )

It is assumed that the diesel doesn't start the 50<sup>th</sup> time.  $P = 0.02$

$$MTBF_{SYS} = 10,000 + (1 - 0.02) \times 10,000 = 19,800 \text{ hours}$$



### 5.2.3 PASSIVE REDUNDANCY, WITH REPAIR

Diesel generators

From Equation 15 ( $MTBF_{SYS} = \frac{MTBF_X + MTTR_X}{(p + (1-p)) \frac{MTTR_X}{MTBF_Y}}$ )

$$MTBF_{SYS} = \frac{MTBF_X + MTTR_X}{(p + (1-p)) \frac{MTTR_X}{MTBF_Y}}$$

$$MTBF_{SYS} \approx \frac{N(MTBF_X + MTTR_X)}{N_{SYSApprox}}$$

Assume MTTR = 40 hours

$$MTBF_{SYS} = \frac{10,000 + 40}{(0.02 + (1 - 0.02)) \frac{40}{10,000}} = 420,000 \text{ hours}$$

### 5.2.4 ACTIVE REDUNDANCY WITHOUT REPAIR

Optic drive operating with a duplicated motor drive where one motor alone will continue to drive the optic when the other motor fails.

From Equation 16 ( $MTBF_{SYS} = \frac{3-2p}{2} MTBF_X$ )

$$MTBF_{SYS} = \frac{3}{2} MTBF_X = \frac{3}{2} (200,000) = 300,000$$

## 6 COMPUTER PROGRAMS

There are many software programs available designed specifically for calculating reliability and availability of systems and for running mission simulations to confirm system configuration availability characteristics. They were originally designed for high-end cost applications such as the defence industries, and are now used more generally including aviation, power generation, oil exploration and transportation, for the analysis of high value processes and systems to minimise production losses and system downtime. As with much engineering software, a mission reliability and availability simulator can be expensive, requiring a considerable investment in resources and training to enable the use of the tools with confidence. The effectiveness of such software or manual calculations, is dependent upon the data with which they are fed and the skill of the operator. The former is the main deficiency when employing them since manufacturers of AtoN systems or components rarely specify their reliability in terms of MTBF.

Although the software can be expensive, many companies make available evaluation packages that can be downloaded free of charge from the Internet. Some of these have only minor restrictions so can be effective for small system investigations and hence AtoN systems. The output displays from such a 'free of charge' evaluation package is illustrated in ANNEX B.

In the absence of published system MTBF figures, they can be calculated on a parts count basis using figures for electronic components in MIL-HDBK-217, a reliability prediction standard originally developed for defence and aerospace industries. Alternatively, assumptions can be made as to the MTBF of a system based upon actual historical data of number of systems deployed over many years.

A further technique has been employed in recent years, using probability theory to analyse a system in terms of the probability of it being in a certain state; in terms of an AtoN for example, whether a navigation light is 'normal' or 'failed'. One technique is known as Bayesian probability and may be useful in the absence of historical or quantitative MTBF data. Bayesian probability enables a statement to be made concerning the belief that a system or component will be in one state depending upon specific knowledge of factors that might affect its



likelihood. Computer software is available which uses this technique representing a system as a Bayesian Network (BN) by defining it in terms of its individual units or components. Although MTBF data is not required a good understanding of the probability of a component being in one state or another is essential, consequently Bayesian networks tend to be used in expert systems.

## 7 QUALITY MANAGEMENT SYSTEMS AND RELIABILITY

The overall reliability and availability of an Aid to Navigation cannot be divorced from the organisations that design, manufacture, install, maintain and operate the AtoN. It is therefore important that these organisations have a quality management system in place to ensure a consistent product and reliable service. The Quality Management System enshrined in the International Standard ISO 9000:2000 (or equivalent) sets out to define the areas that need to be addressed within an organisation to achieve these goals.

The main principles of the ISO 9000 Quality Management system are as follows:

- focus on customers' needs to meet their requirements and expectations;
- lead the organisation establishing a unity of purpose and direction and create an environment that encourages people to commit to and achieve the organisations objectives;
- involve people at all levels encouraging and helping them to develop their abilities;
- manage their activities and resources systematically using defined processes;
- ensure that the processes are interrelated within an overall system;
- encourage the organisation to continually improve their performance and become more effective;
- make decisions based upon the analysis of facts and data;
- work with suppliers and develop a relationship for your mutual benefit.

Implementing and developing these principles will not only ensure a consistent product or service, but will develop the organisation improving efficiency with time.

It should not be assumed that a formal quality management system based upon these principles requires some expensive superstructure supervising the existing organisation. It is often a matter only of rationalising and documenting what is being done already. Guides are available to assist in the setting up of the necessary arrangements and these are adaptable to the needs and resources of the particular authorities.

### 7.1 SPECIFICATIONS

A specification is 'a document and data defining the needs or expectations that are stated, generally implied or obligatory'. It is a key starting point on the road to ensure customer satisfaction. Before drafting a new specification it is advisable to ascertain, from the national and international standards organisation, whether specifications already exist, which will meet the requirements. If it is necessary to draft a new specification it is essential to follow a logical method. The definition of quality covers all aspects of a product or service and it follows that the specification should cover all the features and characteristics. In practice there is considerable diversity in the contents and details given in specifications and it cannot be assumed that specifications for all the products in the same general category will always contain the same requirements from the quality point of view.

### 7.2 SPECIFICATION DATA

It may not be easy to quantify some of the parameters in the specification. One of the most difficult is the definition of the environmental conditions that must be met. Consider, for example, equipment for use on a navigation buoy; acceleration values will vary considerably between different designs of buoys. Ideally tests should be carried out to establish the correct parameters and thus ensure that the specification is neither too strict nor too lax.



It will be noted that the target specification contains the reliability requirements; here again these may be difficult to quantify in the absence of sound data. Figures are available of failure rates of components but it is essential to be satisfied that the conditions, under which the figures were obtained, were relevant to the case under study. There is a need for a data bank of failure rates and reliability based on experience under operational conditions.

### **7.3 MAINTENANCE**

Operating and maintenance departments should be represented on the team engaged on drawing up new specifications and have the opportunity to comment on any revision of existing specifications. In particular, the maintainability, that is how easy the system can be returned to service following a failure, like reliability will have a major effect on availability.

Identification of maintainability requirement during the system life cycle is essential to ensure a successful system design. Factors that affect the overall contribution that maintainability has on system availability will include the geographic location, distance of the AtoN from the maintenance centre and the availability of transportation, resources and spares. However, some of these may be mitigated to a degree by the maintenance practices and procedures employed by the Lighthouse Authority.

In general maintenance can be divided into three types, corrective maintenance, preventive maintenance and inspections.

#### **7.3.1 CORRECTIVE MAINTENANCE**

Corrective maintenance usually involves the repair or replacement of a system component/module to restore the system to a fully operating condition in as short a time as possible. It involves the detection of the fault by the technician, the replacement or repair of the faulty component and finally the testing of the system to prove its satisfactory operation.

#### **7.3.2 PREVENTATIVE MAINTENANCE**

Preventative maintenance is the process by which components or modules are serviced or replaced before they fail in order to support continuous operation without failure. The effect of preventative maintenance on availability can be of major importance since with preventative maintenance system down time can be significantly reduced because it does not include the time to travel to site but only the repair time on site. The schedule for preventative maintenance is based upon reliability statistics and historical data to determine the key system components that are most likely to wear out.

#### **7.3.3 INSPECTIONS**

The primary objective of Inspection is the detection of wear and component degradation and un-revealed failures. If such failures are found, then corrective maintenance would be carried out to rectify the problem.

### **7.4 SELECTION OF A SUPPLIER**

An important factor in achieving reliability and maintainability is the supplier's ability to manage and deliver the project. The factors discussed below should be considered during the tender review process and a scoring system devised to aid comparison between suppliers.

The supplier's design and testing process needs to be assessed and the purchaser needs to look at the company's historical design and test record. They need to ask if the supplier has been able to design equipment in the past without major design faults; is the quality of the design staff that will be working on the contract appropriate and is the quality of the design procedures to be used on the project adequate?

The design phase of the project will also be influenced by its complexity. Is the specification clear and unambiguous, are the requirements novel with untried technologies, are there complex software based control systems? The time schedule permitted for design will also have an impact on the design phase of the project.



Although the production process cannot improve the reliability and maintainability of a product inherent in its design it can have a negative effect. As with the design process similar factors need to be assessed, for example, has the manufacturer produced similar equipment in the past, are the production staff suitably trained and experienced, is there a quality assurance function in the manufacturing process including inspection and testing?

## 7.5 LIGHTHOUSE AUTHORITIES AS SUPPLIERS

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So far this section has dealt with the situation of the Lighthouse Authority as the purchaser, it is as well to consider the Authority as supplier of the service to the mariner. It will have been noted that quality assurance applies equally by definition to services as well as to products and in the role of supplier the Authority has a duty to ensure the quality of the service to the mariner.

It is only rarely that the opportunity arises for the mariner to contribute to the formation of the specification and it is by no means certain that he would wish to do so. Therefore, the Authority is placed in the position of having to write the specification for the service provided as well as applying the necessary quality control.

At first these comments may appear superfluous since all Lighthouse Authorities make great efforts to maintain the highest standards. However, in these days of advanced and complex technology it is worth considering the application of formal quality assurance techniques in order to ensure the best use of money and resources. This is after all only the logical extension of reliability and availability techniques. The setting up of a quality assurance structure, within an Authority, is similar to that required by a manufacturer and is largely a matter of defining responsibility and exercising the necessary control.

The modern trend towards product liability requires serious consideration of formal quality assurance procedures as a form of insurance. The exchange of specifications and standards between Lighthouse Authorities may materially assist in the achievement of quality assurance.

## 8 DEFINITIONS

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The definition of terms used in this Guideline can be found in the International Dictionary of Marine Aids to Navigation (IALA Dictionary) at <http://www.iala-aism.org/wiki/dictionary>.

## 9 ACRONYMS

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A	Availability
AtoN	Aid(s) to Navigation
BS	British Standard
cd	candelas
EN	English
FRT	Failure Response Time
HDBK	Handbook
IALA	International Association of Marine Aids to Navigation and Lighthouse Authorities-AISM
ISO	International Standardization Organisation
IWRAP	IALA Waterway Risk Assessment Program
MIL	Military
MoD	Ministry of Defence (UK)
MFRT	Mean Failure Response Time
MTBF	Mean Time Between Failure
MTTR	Mean Time to Repair
p	probability



PCM	Process Capability Model
R&M	Reliability and Maintainability
SYS	System

## 10 REFERENCES

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- [1] MoD R&M/6/4000/17.A 'A Process Capability Model (PCM) for Reliability and Maintainability (R&M)
- [2] BS EN ISO 9000:2000 'Quality Management Systems – Fundamentals and Vocabulary'

## ANNEX A    PROOF OF FORMULAE

### A 1. BLOCKS IN SERIES (Equation 13)

We shall assume that all blocks stop operating when one of them is out of order and that the equipment has been operating for a very long period of time T and has had a great number of failures, N, comprising  $N_1$  failures of block 1,  $N_2$  failures of block 2 ....  $N_i$  failures of block i.

The total number of failures (N) in a system consisting of i blocks all in series may be expressed as follows:

$$N = \sum_{n=1}^i N_n$$

The MTBF of the system is equal to

$$\frac{T}{N} = \frac{T}{N_1 + N_2 + \dots + N_i}$$

i.e. the total time divided by the total number of failures.

Let  $MTBF_i$  be the MTBF of block number i

$$MTBF_i = \frac{T}{N_i}$$

then:

$$\frac{N}{T} = \frac{N_1}{T} + \frac{N_2}{T} + \dots + \frac{N_i}{T}$$

that is to say:

$$\frac{1}{MTBF_{SYS}} = \frac{1}{MTBF_X} + \frac{1}{MTBF_Y} + \dots + \frac{1}{MTBF_i}$$

The invert of the MTBF of a system with all blocks in series is equal to the sum of inverses of the MTBF of each block.

A very popular demonstration of this formula is given hereafter:

The probability that two independent events occur during a trial is equal to the product of the individual probability of each event. Then the probability that a system with i blocks in series does not fail during a period of time t is equal to

$$R(t) = e^{-(\lambda_x t)} + e^{-(\lambda_y t)} + \dots + e^{-(\lambda_i t)}$$

If each block has a constant failure rate.

It results from the property of the exponential that

$$R(t) = e^{-\sum_{n=1}^i (\lambda_n t)}$$

so, the total failure rate  $\lambda_{SYS}$  of the system is equal to

$$\lambda_{SYS} = \sum_{n=1}^i \lambda_n$$

Note: With the same notations as before:

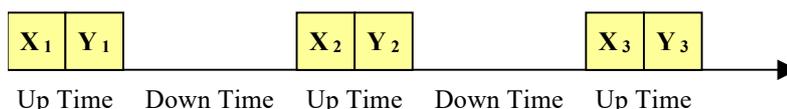
$$MTTR_{SYS} = \frac{\sum_{n=1}^i N_n MTTR_n}{N} = \frac{\sum_{n=1}^i \frac{T}{MTBF_n} MTTR_n}{\frac{T}{MTBF_{SYS}}}$$

And thus:

$$MTTR_{SYS} = MTBF_{SYS} \sum_{n=1}^i \frac{MTTR_n}{MTBF_n}$$

## A 2. PASSIVE REDUNDANCY WITHOUT REPAIR (Equation 14)

The succession of up times and down times is as follows:



By definition

$$MTBF_{SYS} = \frac{1}{n} \sum_{n=1}^i (X_n + Y_n)$$

where  $n$  is large and is the total number of failures of the system. So:

$$MTBF_{SYS} = \frac{1}{n} \sum_{n=1}^i X_n + \frac{1}{n} \sum_{n=1}^i Y_n = MTBF_X + MTBF_Y$$

If  $Y$  has a probability equal to  $p$ , not to start operating correctly when  $X$  fails, then it is only in  $100 \times (1-p) \%$  of the cases that the operating time of  $Y$  will be added to that of  $X$ .

It follows that:

$$MTBF_{SYS} = MTBF_X + (1-p)MTBF_Y$$

## A 3. PASSIVE REDUNDANCY, WITH REPAIR (Equation 15)

Let  $n$  be the number of failures of  $X$  during a long period of time equal to  $n \times (MTBF_X + MTTR_X)$ .

The block  $Y$  will have to operate during the down times of  $X$ . The total down time during the period in question will be equal to  $N_X = MTTR_X$ , and there will be on average a failure of  $Y$ , (and so of the system) every  $MTBF_Y$  hours (Counted on the basis of the down time of  $X$ ).

Then the number of failures  $N_{SYS}$  of the system  $S$  will be very close to:

$$N_{SYSApprox} = \frac{N_X(MTTR_X)}{MTBF_Y}$$

By definition:

$$MTBF_{SYS} = \frac{N_X(MTBF_X + MTTR_X)}{N_{SYS}} \approx \frac{N_X(MTBF_X + MTTR_X)}{N_{SYSApprox}}$$

Thus, it follows that if  $X$  and  $Y$  have the same statistical characteristics:



$$MTBF_{SYS} \approx \frac{MTBF_X^2}{MTTR_X} + MTBF_X$$

If it is assumed that there is a probability  $p$  that the block Y does not start when activated, then the same demonstration as before leads to the following formula:

$$N_{SYSApprox} = Np + \frac{N(1-p)MTTR_x}{MTBF_Y}$$

and

$$MTBF_{SYS} \approx \frac{N(MTBF_X + MTTR_X)}{N_{SYSApprox}} = \frac{MTBF_X + MTTR_X}{(p + (1-p)) \frac{MTTR_X}{MTBF_Y}}$$

#### A 4. ACTIVE REDUNDANCY WITHOUT REPAIR (Equation 16)

If  $p = 0$  the proof is rather simple:

X and Y being statistically independent then  $P(S \leq t) = P(X \leq t) \times P(Y \leq t)$

If in addition X and Y are identical with a constant failure rate then:

$$P(S \leq t) = (1 - e^{-\lambda t})(1 - e^{-\lambda t}) = 1 - 2e^{-\lambda t} + e^{-2\lambda t}$$

The reliability function  $R(t)$  is then equal to:

$$R(t) = 2e^{-\lambda t} + e^{-2\lambda t}$$

and:

$$MTBF_{SYS} = \int_0^{\infty} (2e^{-\lambda t} - e^{-2\lambda t}) dt = \frac{2e^0}{\lambda} - \frac{e^0}{2\lambda} = \frac{3}{2} \times \frac{1}{\lambda}$$

where:

$$\frac{1}{\lambda} = MTBF$$

That is to say:

$$MTBF_{SYS} = \frac{3}{2} MTBF$$

If  $d \neq 0$  the proof is more complicated and is only given hereafter for those familiar with the theory of probability.

Let  $FX$  and  $FY$  be respectively the states where X and Y are in good running order at  $t$ .

The event 'the failure of one element does not cause that of the other' will be quoted as  $FB$ .

$$\overline{FX}, \overline{FY} \text{ and } \overline{FB}$$

Let  $\overline{FX}$ ,  $\overline{FY}$  and  $\overline{FB}$  be the events complementary to  $FX$ ,  $FY$   $FB$  respectively.

Then by applying the theorem of 'total probabilities' it follows that the probability  $P(S \leq t)$  of a failure of the system occurring before  $t$  is given by:

$$P(S \leq t) = P(FB)P(\overline{FX} \cup \overline{FY}) + P(\overline{FB})P(\overline{FX} \cap \overline{FY})$$

$$P(\overline{FB})[P(\overline{FX}) + P(\overline{FY}) - P(\overline{FX})P(\overline{FY})] + P(FB)P(\overline{FX})P(\overline{FY})$$

$$P(\overline{FB}) = p \text{ and } P(FB) = (1 - p)$$

$$P(\overline{FX}) = (1 - e^{-\lambda t}) \text{ and } P(\overline{FY}) = (1 - e^{-\lambda t})$$

$$\int_0^{\infty} P(S \leq t) dt = MTBF_{SYS}$$

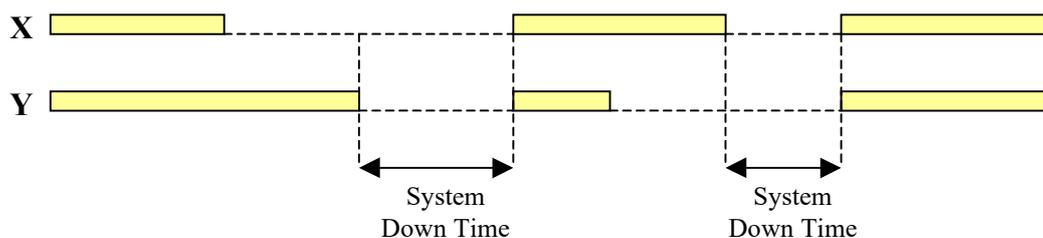
a simple calculation shows that:

$$(MTBF_{SYS} = \frac{3-2p}{2} MTBF_X)$$

It should be noted that the same method would permit calculation of  $MTBF_{SYS}$  when the 2 blocks do not have the same failure rate.

## A 5. ACTIVE REDUNDANCY WITH REPAIR OF A FAILED ITEM (Equation 17)

The operating times may be represented as follows:



Calculating  $IS = (1 - A_{SYS})$  where  $A_{SYS}$  is the availability of the system

$$IS = \frac{MTTR_X}{(MTBF_X + MTTR_X)} \times \frac{MTTR_Y}{(MTBF_Y + MTTR_Y)}$$

As, by definition:

$$IS = \frac{MTTR_{SYS}}{(MTBF_{SYS} + MTTR_{SYS})}$$

by looking at the operating diagram one can notice that the model for calculating  $MTTR_{SYS}$  is equivalent to one with blocks in series where time to repair is substituted for operating time (up time) and vice versa and where it is assumed that a block continues to operate even when the other has failed.

Having done that, the basic hypotheses are not preserved since mean time to repair is greater than MTBF and then the demonstration given in section 2.1 can't be applied.

However, one can show that the following formula is still valid

$$\frac{1}{MTTR_{SYS}} = \frac{1}{MTTR_X} + \frac{1}{MTTR_Y}$$

and so:

$$MTBF_{SYS} = \frac{1}{\left(\frac{1}{MTTR_X} + \frac{1}{MTTR_Y}\right)} \left[ \frac{1-IS}{IS} \right]$$

If X and Y are identical, then:

$$MTTR_{SYS} = \frac{MTTR_X}{2}$$

and

$$MTBF_{SYS} = \frac{MTTR_X}{2} \left[ \left[ 1 + \frac{MTBF_X}{MTTR_X} \right]^2 - 1 \right]$$

The above formula can be generalized to the case where  $i$  blocks are in active redundancy.



If the  $i$  blocks are identical the formulae are:

$$MTTR_{SYS} = \frac{MTTR_X}{i}$$

and

$$MTBF_{SYS} = \frac{MTTR_X}{i} \left[ \left[ 1 + \frac{MTBF_X}{MTTR_X} \right]^i - 1 \right]$$

Notes

- 1 A more efficient policy being in general applied then the whole system S has failed; the above  $MTTR_{SYS}$  should be considered only as a mean to calculate  $MTBF_{SYS}$ .
- 2 Under the basic hypothesis it can be shown that the probability that the system S still operates at time  $t$  is close to  $e^{-\frac{t}{MTBF_{SYS}}}$

## A 6. RELATIONSHIP BETWEEN PREVENTIVE MAINTENANCE AND RELIABILITY

The proof is quite easy for those familiar with the theory of probability:

By definition:

$$P_0 = \int_0^t F(t) dt$$

Let  $P_0$  be the probability that the equipment fails after having been replaced  $q$  times, then:

$$P_q = P_0(1 - P_0)^q$$

Since  $q$  components have been changed after having been operating correctly during  $T$  hours and the  $q + 1$  component failed before  $T$  hours of operation.

$$MTBF_0 = \frac{\int_0^T t f(t) dt}{\int_0^T f(t) dt}$$

and

$$\sum_0^{\infty} (iT + MTBF_0) P_i = MTBF_0 \sum_0^{\infty} P_i + T \sum_0^{\infty} iP_i$$

In this expression

$$\sum_0^{\infty} P_i = 1$$

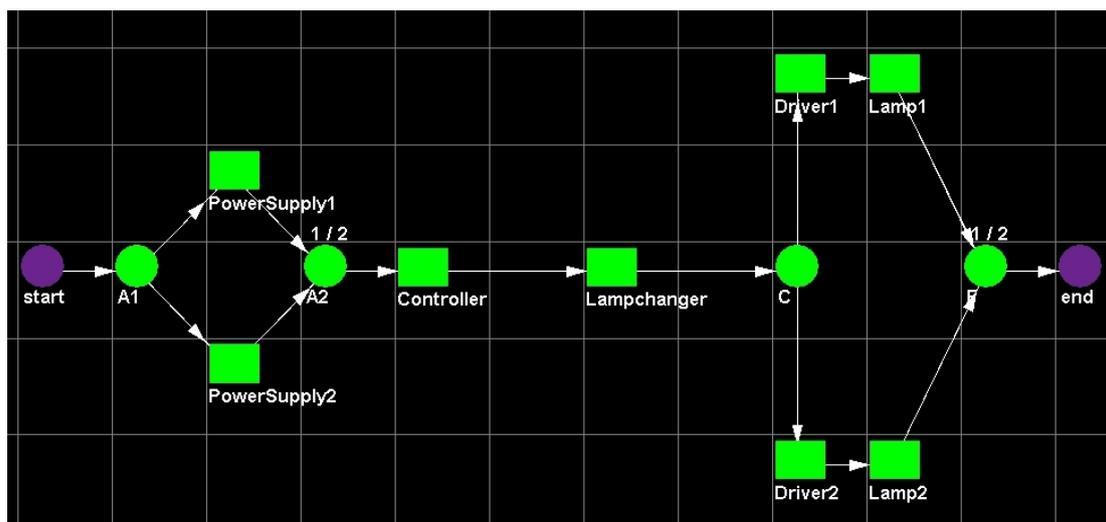
and

$$\sum_0^{\infty} iP_i = P_0 \sum_0^{\infty} i(1 - P_0)^i = \frac{1 - P_0}{P_0}$$

It follows that:

$$MTBF = MTBF_0 + T \left[ \frac{1 - P_0}{P_0} \right]$$

## ANNEX B TYPICAL GRAPHICAL REPORT FROM A RELIABILITY SOFTWARE PACKAGE



**Figure 13 Reliability Block Diagram of Typical AtoN**

Final Results

Results from 10 run(s):

PARAMETER	MEAN	MIN	MAX	ST DEV
Ao	0.993672081	0.984393892	1.000000000	0.005140701
MTBDE	>7796.353529	3281.312976	>10000.000000	n/a
MDT (7 runs)	52.828292	51.294133	54.931356	1.212934
MTBM	763.312332	555.555556	1236.442327	194.017785
MRT	19.426566	13.075631	31.228104	5.151837
% Green Time	96.928465	94.768458	98.375058	1.058740
% Yellow Time	2.438743	0.540328	4.175222	1.016958
% Red Time	0.632792	0.000000	1.560611	0.514070
System Failures	1.200000	0	3	0.979796

$R(t=10000.000000) = 0.300000$

**Figure 14 Reliability Report Following Simulation**