GUIDELINE

G1135

DETERMINATION AND CALCULATION OF EFFECTIVE INTENSITY

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<table>
<thead>
<tr>
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<th>Details</th>
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<tbody>
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</tr>
</tbody>
</table>
CONTENTS

1. INTRODUCTION ................................................................................................................................... 4
2. DESCRIPTION OF EFFECTIVE INTENSITY OF A RHYTHMIC LIGHT............................................................... 4
3. EVALUATION OF EFFECTIVE INTENSITY..................................................................................................... 5
  3.1. Modified Allard Method .......................................................................................................................... 5
    3.1.1. Continuous Time Version ........................................................................................................................... 5
    3.1.2. Discrete-time Version ......................................................................................................................................... 6
4. CONCLUSIONS ..................................................................................................................................... 7
5. REFERENCES ........................................................................................................................................ 8

ANNEX A Computational Considerations of the Modified Allard Method ................................................................. 9

List of Figures

Figure 1 Graphical representation of the Modified Allard Method Visual System Response Function, q(t), for different values of a. Negative values of t result in a value of 0 for q(t). ................................................. 6
Figure 2 Flash profile with resultant convolution using the Modified Allard Method ............................................. 7
Figure 3 Plot of intensity against time, I(t), and visual impulse function, q(t) ......................................................... 9
Figure 4 Histograms of flash profile, I(t), and the visual response function, q(t)....................................................... 9
Figure 5 Figure 5 Convolution at t = 0 ........................................................................................................ 10
Figure 6 Figure 6 Convolution at t = 1 ........................................................................................................ 10
Figure 7 Figure 7 Convolution at t = 2 ........................................................................................................ 11
Figure 8 Figure 8 Convolution at t = 3 ........................................................................................................ 11
Figure 9 Figure 9 Convolution at t = 9 showing a maximum value at t = 7 ..................................................... 12
Figure 10 Figure 10 Continuous graph of flash profile I(t) and convolution product ...................................... 12

List of Equations

Equation 1 Blondel-Rey Expression for the Effective Intensity .............................................................................. 4
Equation 2 Modified Allard Method .................................................................................................................. 5
Equation 3 Visual System Response Function .................................................................................................. 5
Equation 4 Discrete Convolution Equation (Step 1 of Modified Allard Method) ................................................. 6
Equation 5 Effective intensity from the discrete convolution equation (Step 2 of Modified Allard Method) .. 7
1. **INTRODUCTION**

The scope of this document is all flashing marine aid-to-navigation signal lights with a flash duration of five seconds or less. Lights with a flash duration of greater than five seconds may be considered as continuous or fixed lights.

The purpose of this document is to describe how to calculate the effective intensity of a given flash of light when viewed at the IALA defined illumination threshold for visual signalling. In the past, effective intensity models have been based on the achromatic threshold, which does not necessarily model the human visual system response accurately at signal illumination levels used for visual signalling.

Nevertheless, the Modified Allard Method described below has been demonstrated to match observations well at the visual signalling illuminance threshold, despite its origins being for the calculation of effective intensity at achromatic threshold.

2. **DESCRIPTION OF EFFECTIVE INTENSITY OF A RYTHMIC LIGHT**

The range at which an observer may just see a light flash may be described in terms of a single parameter which is called the ‘effective intensity’ of the flash. The eye does not analyse the variations of the luminous flux incident upon it during the course of a brief flash but reacts to the total visual impression of the flash of light. In particular, when the flash can just be seen it is possible to obtain a quantitative measure of the effectiveness of its light by comparing it with a steady light, which is also just seen under the same conditions at the same range, and by the same observer. Sufficient consistency is obtained in such observations to permit the evaluation of effective intensity of the flash as the intensity of the fixed light, which is its equivalent for detection at the threshold of visual perception (achromatic threshold). In this document, the recommended method of evaluating the effective intensity for various flash forms (distributions of luminous intensity with time) will be considered. The effective intensity is defined by the equivalence of fixed and flashing lights at threshold levels, and levels above threshold are not considered. Unless otherwise stated, the evaluations are for single flashes, with the lowest effective intensity of the flashes in a character defining the nominal range of that light.

To permit the use of the Modified Allard Method for evaluation of effective intensity which shall be simple, universally applicable and of sufficient accuracy for practical purposes of marine aid-to-navigation provision, the other conditions of observation have been restricted to certain standard reference values, which have been chosen to represent typical average conditions for marine observation of lights:

1. Young observer with normal vision;
2. Subtense angle of light source at the eye of the observer ≤ 1′;

In general, the Modified Allard Method makes use of time constants of the visual system, denoted by $\tau$. The constant is the same as the more familiar time-constant $\tau$ of the Blondel-Rey expression for the effective intensity $I_e$ of flashes of rectangular form:

$$\frac{I_e}{I_0} = \frac{t}{\tau + t}$$

*Equation 1  Blondel-Rey Expression for the Effective Intensity*

Where:

- $I_e$ is the effective intensity (cd)
- $I_0$ is the peak intensity (cd)
- $t$ is the length of the rectangular flash (s)
In general, the time-constants are dependent on the colour of the light exhibited, on the level of background luminance against which the light is seen, and on the angular subtense of the light source at the eye of the observer.

Under the reference conditions stated above: for both daytime and night-time observations, it is recommended that the value of \( a \) be taken equal to 0.1 second for all signal colours, except blue, which shall be taken equal to 0.2 second at night.

The value of \( a \) has changed from previous editions of the effective intensity recommendations to make the Modified Allard Method fit closer to observations.

### 3. EVALUATION OF EFFECTIVE INTENSITY

The determination of effective intensity for any given flash proceeds from knowledge of the variation of the instantaneous luminous intensity with time. It is usually desirable both to determine the form of this variation and to scale the curve so that the ordinates are the values of luminous intensity at each instant. Photometric measurements of luminous intensity and of the distribution of luminous intensity with time have been described in IALA Recommendation E200-3, and the difficulties and limitations inherent in them have been discussed.

#### 3.1. MODIFIED ALLARD METHOD

**3.1.1. CONTINUOUS TIME VERSION**

In the Modified Allard Method, the effective intensity, \( I_e \), of a finite length flash is determined by the maximum value of the convolution result between the flash profile and the visual system response function. Thus,

\[
I_e = \max_t \left\{ \int_{-\infty}^{+\infty} I(t - t') \cdot q(t') \, dt' \right\}
\]

**Equation 2**  
**Modified Allard Method**

Where:

- \( I(t) \) is the instantaneous luminous intensity of the flash at a time \( t \).
- \( q(t) \) is the visual system response function.

The visual system response function, \( q(t) \), is determined by:

\[
q(t) = \begin{cases} 
\frac{a}{(a + t)^2} & \text{for } t \geq 0 \\
0 & \text{for } t < 0
\end{cases}
\]

**Equation 3**  
**Visual System Response Function**

Where:

- \( a = \begin{cases} 
0.1 \text{ s} & \text{for all signal colours except blue at night} \\
0.2 \text{ s} & \text{for blue signal colour at night}
\end{cases} \)

Figure 1 shows the Modified Allard visual system response function, \( q(t) \), plotted as a function of time.
3.1.2. **DISCRETE-TIME VERSION**

It can be shown that the continuous time version of Modified Allard Method can be utilised for discrete-time applications, such as photometric samples taken at **regular intervals**. The methods of measuring are discussed in IALA Recommendation E200-3. Once a set of samples has been obtained, either by measurement or by synthesis in a spreadsheet, the first step of calculating the effective intensity of the flash can be achieved using the following equation.

\[
i(t_j) = \Delta t \left( \frac{I(t_0)q(t_j - t_0)}{2} + \sum_{k=1}^{N-1} I(t_k)q(t_j - t_k) + \frac{I(t_N)q(t_j - t_N)}{2} \right)
\]

**Equation 4**  *Discrete Convolution Equation (Step 1 of Modified Allard Method)*

Where:

- \( I(t) \) is the sampled data at time \( t_0, t_1, t_2, ..., t_N \) over the entire flash duration
- \( N \) is the number of data points
- \( t_k \) is the time of the \( k \)-th data point
- \( t_j \) is the time of the \( j \)-th data point
- \( \Delta t \) is \( \frac{t_N - t_0}{N} \) (the time interval between samples)
- \( q(t) \) as defined in Equation 3.

Equation 4 makes a few assumptions in order to simplify the calculation. The flash being considered should exist for a positive value of \( t \), and at the limits of the dataset, the flash should be considered extinguished. Also, being a convolution function, the number of iterations needed to calculate the result increases exponentially with the length of the dataset. A long duration flash or a flash sampled at a high rate will result in a slower computation. However, the value of \( \Delta t \) (the interval between samples) should be sufficiently small to ensure that the flash profiles is accurately captured (including any pulse width modulation used).

If using a spreadsheet, the ‘SUMPRODUCT’ function may be used to convolve \( I(i) \) and reverse \( q(i) \) functions in order to determine the effective intensity of a measured flash profile. Discrete time steps for both functions should be the same. Figure 2 shows graphically the measured flash (dark blue) and the \( q(i) \) function (purple) used in an example. The resulting convolution product, \( i(t) \), is shown in red.
The effective intensity value is the maximum of the convolution, such that:

\[ I_e = \max_{t_f} \{ q(t_f) \} \]

Equation 5  **Effective intensity from the discrete convolution equation (Step 2 of Modified Allard Method)**

In Figure 2, it can be see that the maximum value of the convolution product has a maximum value of approximately 120,000 cd, so this would be the effective intensity of the flash shown in this figure.

The advantages of the Modified Allard method are:

- It is mathematically equivalent to the Blondel-Rey equation for rectangular pulses;
- It is suitable for train of pulses as validated by the visual experimental data [1] as well as by computational analysis;

More information on the computational considerations of applying the Modified Allard Method is shown in ANNEX A.

4. **CONCLUSIONS**

- The Modified Allard method is the method recommended for determining the effective intensity of a marine AtoN signal light of any flash profile or multiple flash profiles at any repetition rate;
- The Blondel-Rey method, Equation 1, may be used to determine the effective intensity of a single flash of a marine AtoN signal light **providing** the flash profile is rectangular. It should not be used for repeated flashes that flash at a rate greater than 60 flashes per minute;
- If, and only if, it is impossible to measure the variation of instantaneous intensity with time, an estimation of effective intensity may be calculated from the Blondel-Rey formula, Equation 1, using values of \( I_o \) and \( t \) calculated by methods outlined in IALA Recommendation E200-5\(^1\);
- It should be recognised that the Modified Allard Method is not perfect, and that IALA members are requested to submit information that would enable the effective intensity model to be improved.

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\(^1\) To date (December 2017) the E-200 series of Recommendations is under review. It is likely that the details contained in Recommendation E-200-5 are removed to a new Guideline before 2022.
• In addition, IALA members should consider improvements to the effective intensity calculation and are requested to submit those to IALA.

5. REFERENCES

ANNEX A  COMPUTATIONAL CONSIDERATIONS OF THE MODIFIED ALLARD METHOD

The Modified Allard method of calculating effective intensity is achieved by mathematical convolution. This process can better be described by considering the discrete data resulting from a measurement of the variation intensity over time with a digital recording device. Figure 3 is a typical flash profile from a rotating beacon and, with it, the visual impulse function.

![Figure 3](image)

**Figure 3**  Plot of intensity against time, I(t), and visual impulse function, q(t)

The squares marked on the flash plot are instances in time when the instantaneous intensity was recorded digitally. Both flash profile and visual impulse function can be shown as discrete values by a histogram.

![Histograms](image)

**Figure 4**  Histograms of flash profile, I(t), and the visual response function, q(t)

The convolution is achieved by stepping the reverse visual impulse function past the flash profile taking the sum product at each step as follows:
At the first step, the value of q₁ in the visual impulse function is multiplied by the value of I₁ in the flash profile. This product is multiplied by the time increment in seconds to give the convolved value for t=1.
At t=2, the value of $q_1$ is multiplied by the value of $I_2$, then the value of $q_2$ is multiplied by $I_1$. Both products are then added together and multiplied by the time increment. The result is the convolved value for $t=2$.

At t=3, the value of $q_1$ is multiplied by the value of $I_3$, the value of $q_2$ is multiplied by $I_2$ and the value of $q_3$ is multiplied by $I_1$. These three products are then added together and multiplied by the time increment to obtain the resultant convolved value for $t=3$. 
As this process is continued through steps 0 to 9 it is possible to see the convolution plot emerging:

![Convolution Plot](image)

**Figure 9**  **Figure 5 Convolution at t = 9 showing a maximum value at t = 7**

Although rather crude, the histograms show the convolution process in discrete format. Reverting to the continuous format, the peak value of the convolution can be taken as the effective intensity value.

![Continuous Graph](image)

**Figure 10**  **Figure 6 Continuous graph of flash profile I(t) and convolution product**

The discrete values of the flash profile, reversed visual impulse function and time increments can be entered into a spreadsheet. The SUMPRODUCT function may be employed to give a value of the convolution at each time increment. Of the resultant convolved values shown at each time increment, the maximum value should be taken to obtain the effective intensity.