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IALA GUIDELINE

G1135
DETERMINATION AND CALCULATION OF EFFECTIVE INTENSITY

Edition 3.1

June 2022

urn:mrn:iala:pub:g1135:ed3.1

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Association Internationale de Signalisation Maritime

DOCUMENT REVISION

Revisions to this document are to be noted in the table prior to the issue of a revised document.

Date	Details	Approval
December 2017	First issue	Council 65
December 2020	Edition 2.0 Added Section 3.2 on estimating effective intensity. Added a section on using Fast Fourier Transform in Annex A. Added pre-calculated peak-to-effective intensity tables in Annex B.	Council 72
June 2022	Edition 3.0 Revision to Annex B.	Council 75
July 2022	Edition 3.1 Editorial corrections.	

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1. INTRODUCTION

The scope of this document is all flashing Marine Aid to Navigation signal lights with a flash duration of five seconds or less. Lights with a flash duration of greater than five seconds may be considered as continuous or fixed lights.

The purpose of this document is to describe how to calculate the effective intensity of a given flash of light when viewed at the IALA defined illumination threshold for visual signalling. In the past, effective intensity models have been based on the achromatic threshold, which does not necessarily model the human visual system response accurately at signal illumination levels used for visual signalling.

Nevertheless, the Modified Allard Method described below has been demonstrated to match observations well at the visual signalling illuminance threshold, despite its origins being for the calculation of effective intensity at achromatic threshold.

2. DESCRIPTION OF EFFECTIVE INTENSITY OF A RHYTHMIC LIGHT

The range at which an observer may just see a light flash may be described in terms of a single parameter which is called the 'effective intensity' of the flash. The eye does not analyse the variations of the luminous flux incident upon it during the course of a brief flash but reacts to the total visual impression of the flash of light. In particular, when the flash can just be seen it is possible to obtain a quantitative measure of the effectiveness of its light by comparing it with a steady light, which is also just seen under the same conditions at the same range, and by the same observer. Sufficient consistency is obtained in such observations to permit the evaluation of effective intensity of the flash as the intensity of the fixed light, which is its equivalent for detection at the threshold of visual perception (achromatic threshold). In this document, the recommended method of evaluating the effective intensity for various flash forms (distributions of luminous intensity with time) will be considered. The effective intensity is defined by the equivalence of fixed and flashing lights at threshold levels, and levels above threshold are not considered. Unless otherwise stated, the evaluations are for single flashes, with the lowest effective intensity of the flashes in a character defining the nominal range of that light.

To permit the use of the Modified Allard Method for evaluation of effective intensity for Marine Aid-to-Navigation purposes, a number of assumptions are made. Namely,

1 Young observer with normal vision;

2 Subtense angle of light source at the eye of the observer ≤ 1′;

3 Applies to all light colours.

In general, the Modified Allard Method makes use of time constants of the visual system, denoted by a. The constant is the same as the more familiar time-constant a of the Blondel-Rey expression for the effective intensity I_e of flashes of rectangular form as shown in Figure 1:

$$I_e = I_o \frac{t}{a+t}$$

Equation 1 Blondel-Rey Expression for the Effective Intensity

Where:

I_e is the effective intensity (cd)

I_o is the peak intensity (cd)

t is the length of the rectangular flash (s)

a is the visual time constant (s).

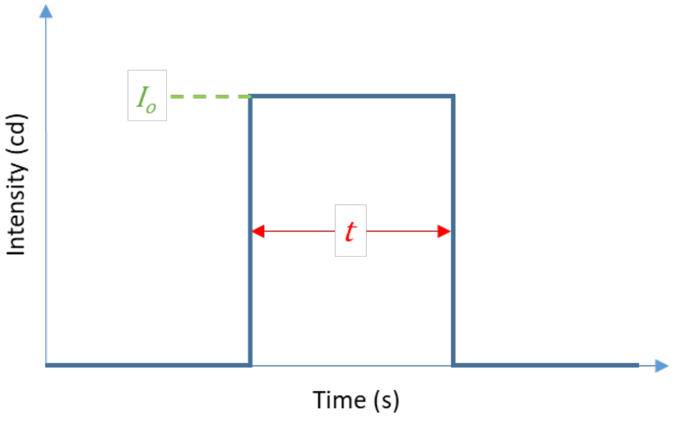


Figure 1 Rectangular flash shape

In general, the time-constants are dependent on the colour of the light exhibited, on the level of background luminance against which the light is seen, and on the angular subtense of the light source at the eye of the observer.

Further to the assumptions stated above: for both daytime and night-time observations, it is recommended that the value of a be taken equal to 0.1 seconds for all signal colours, except blue, which shall be taken equal to 0.2 seconds at night.

The value of a has changed from previous editions of the effective intensity recommendations to make the Modified Allard Method fit closer to observations.

3. EVALUATION OF EFFECTIVE INTENSITY

The determination of effective intensity for any given flash proceeds from knowledge of the variation of the instantaneous luminous intensity with time. It is usually desirable both to determine the form of this variation and to scale the curve so that the ordinates are the values of luminous intensity at each instant. Photometric measurements of luminous intensity and of the distribution of luminous intensity with time have been described in IALA Recommendation *E200-3*, and the difficulties and limitations inherent in them have been discussed.

3.1. MODIFIED ALLARD METHOD

3.1.1. CONTINUOUS TIME VERSION

In the Modified Allard Method, the effective intensity, I_e, of a finite length flash is determined by the maximum value of the convolution result between the flash profile and the visual system response function. Thus,

$$I_e = \max_t \left\{ \int_{-\infty}^{+\infty} I(t-t') \cdot q(t')\, dt' \right\}$$

Equation 2 Modified Allard Method

Where:

$I(t)$ is the instantaneous luminous intensity of the flash at a time t,

$q(t)$ is the visual system response function

The visual system response function, $q(t)$, is determined by:

$$q(t) = \begin{cases} \dfrac{a}{(a+t)^2} & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

Equation 3 Visual System Response Function

Where:

$$a = \begin{cases} 0.1\ \mathrm{s} & \text{for all signal colours except blue at night.} \\ 0.2\ \mathrm{s} & \text{for blue signal colour at night.} \end{cases}$$

Figure 2 shows the Modified Allard visual system response function, $q(t)$, plotted as a function of time.

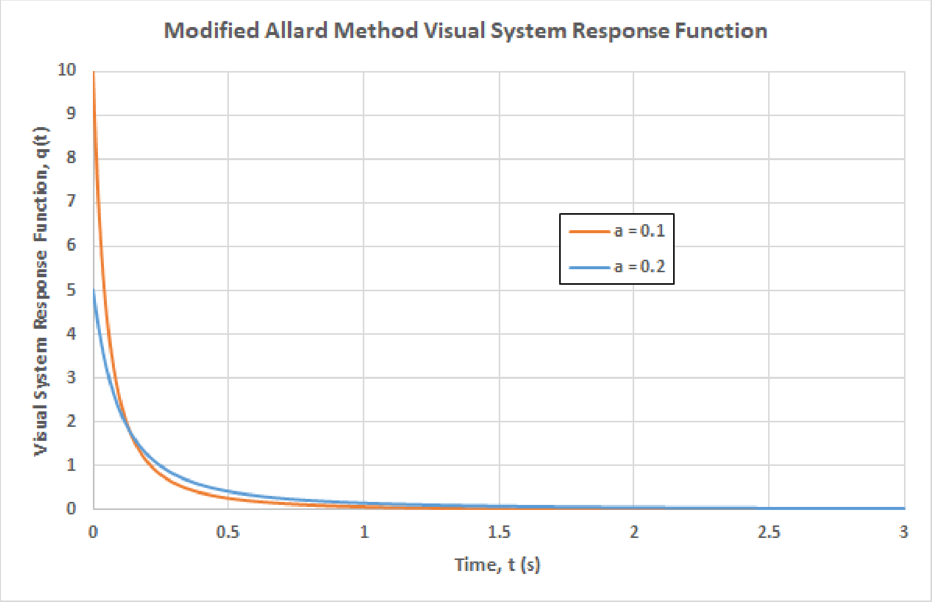


Figure 2 Graphical representation of the Modified Allard Method Visual System Response Function, q(t), for different values of a. Negative values of t result in a value of 0 for q(t).

3.1.2. DISCRETE-TIME VERSION

It can be shown that the continuous time version of Modified Allard Method can be utilised for discrete-time applications, such as photometric samples taken at *regular intervals*. The methods of measuring are discussed in IALA Recommendation *E200-3*. Once a set of samples has been obtained, either by measurement or by synthesis in a spreadsheet, the first step of calculating the effective intensity of the flash can be achieved using the following equation.

$$i(t_j) = \Delta t \left(\frac{I(t_0)q(t_j - t_0)}{2} + \sum_{k=1}^{N-1} I(t_k)q(t_j - t_k) + \frac{I(t_N)q(t_j - t_N)}{2} \right)$$

Equation 4 Discrete Convolution Equation (Step 1 of Modified Allard Method)

Where:

$I(t)$ is the sampled data at time $t_0, t_1, t_2, \ldots, t_N$ over the entire flash duration

N is the number of data points

t_k is the time of the k-th data point

t_j is the time of the j-th data point

Δt is $\frac{t_N - t_0}{N}$ (the time interval between samples)

$q(t)$ as defined in Equation 3

Equation 4 makes a few assumptions in order to simplify the calculation. The flash being considered should exist for a positive value of t, and at the limits of the dataset, the flash should be considered extinguished. Also, being a convolution function, the number of iterations needed to calculate the result increases exponentially with the length of the dataset. A long duration flash or a flash sampled at a high rate will result in a slower computation. However, the value of Δt (the interval between samples) should be sufficiently small to ensure that the flash profiles is accurately captured (including any pulse width modulation used).

If using a spreadsheet, the 'SUMPRODUCT' function may be used to convolve $I(t)$ and reverse $q(t)$ functions in order to determine the effective intensity of a measured flash profile. Discrete time steps for both functions should be the same. Figure 2 shows graphically the measured flash (dark blue) and the $q(t)$ function (purple) used in an example. The resulting convolution product, $i(t)$, is show in red.

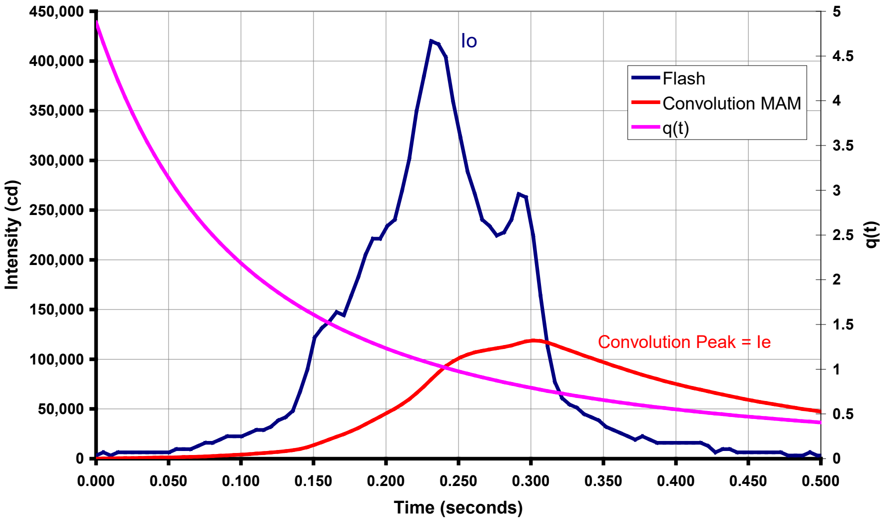


Figure 3 Flash profile with resultant convolution using the Modified Allard Method

The effective intensity value is the maximum of the convolution, such that:

$$I_e = \max_{t_j} \{ i(t_j) \}$$

Equation 5 Effective intensity from the discrete convolution equation (Step 2 of Modified Allard Method)

In Figure 2, it can be seen that the maximum value of the convolution product has a maximum value of approximately 120 000 cd, so this would be the effective intensity of the flash shown in this figure.

The advantages of the Modified Allard method are:

- It is mathematically equivalent to the Blondel-Rey equation for rectangular pulses;
- It is suitable for train of pulses as validated by the visual experimental data [1] as well as by computational analysis;

More information on the computational considerations of applying the Modified Allard Method is shown in ANNEX A.

3.2. ESTIMATING EFFECTIVE INTENSITY

It is not always possible to obtain a measured flash profile for the calculation of effective intensity. In this case, it is possible to use the tables provided in ANNEX B containing pre-calculated factors to convert peak intensity to effective intensity for a number of common flash shapes. Such tables can be useful during the design stage of an AtoN to ensure that the light meets the navigation requirements, as described in Guideline *G1148 Determination of Required Luminous Intensity for Marine Signal Lights.*

Two sets of tables are given to allow for the different values of the visual constant, a, and the correct table should be used for the signal light colour and viewing conditions.

The flash lengths are given by the duration of the flash at 50% of peak intensity (Full Width Half Maximum, FWHM), so care must be taken since the flash duration is not necessarily the duration of the complete flash.

For flash lengths not listed in the table, linear interpolation using the values in the tables can be used to calculate a suitable factor.

3.2.1. PULSE-WIDTH MODULATION

In modern LED lanterns, pulse-width modulation (PWM) is often used to control the perceived intensity of the light. The modulation technique switches the light fully on and fully off at a frequency higher than can be perceived by the human eye. The perceived intensity of the light is dependent on the switching duty cycle – the ratio of on time to off time during a single period. In simplistic terms, if a light is switching such that it is on for 75% of the switching period, then the light will appear 75% the intensity of a fully-on light. An example of this is shown in Figure 4, where the blue trace shows the intensity of the light being switched using PWM, and the orange trace shows how the light would be perceived. Despite the light being at full power for some of the time, the repetition rate and duty cycle causes the light to be seen as a steady lower intensity light.

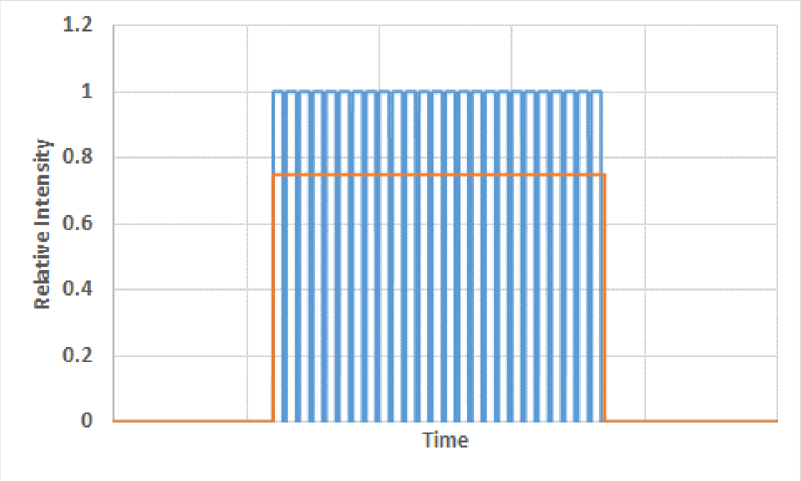


Figure 4 Setting the intensity of a rectangular flash using PWM

More complex use of PWM can be used to create different flash shapes, as shown in Figure 5. This shows how the PWM shown in blue is used to create the trapezoid shape shown in orange. As the PWM duty cycle increases, so does the perceived light intensity.

In Figure 5, the flash peak intensity is the same as that of the light that is being switched on and off. In this case, it is possible to apply the peak-to-effective intensity factors shown in Annex B directly by selecting the flash shape that the PWM creates. The peak intensity is the peak intensity of the light being modulated.

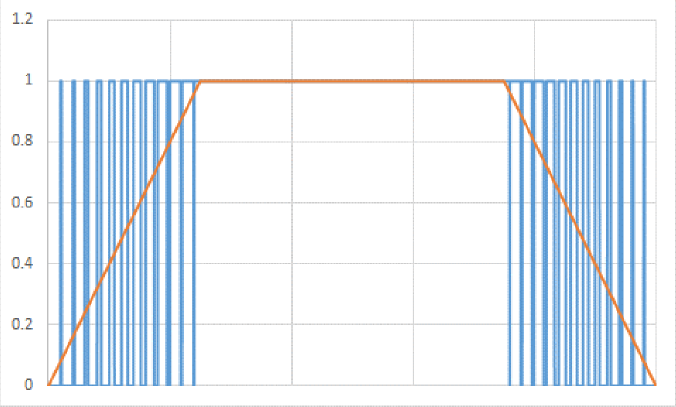


Figure 5 Creating flash shapes with PWM

However, this is not always the case. It is possible that the peak of the flash shape is not the same as that of the peak intensity of the light. Such an example is shown in Figure 6.

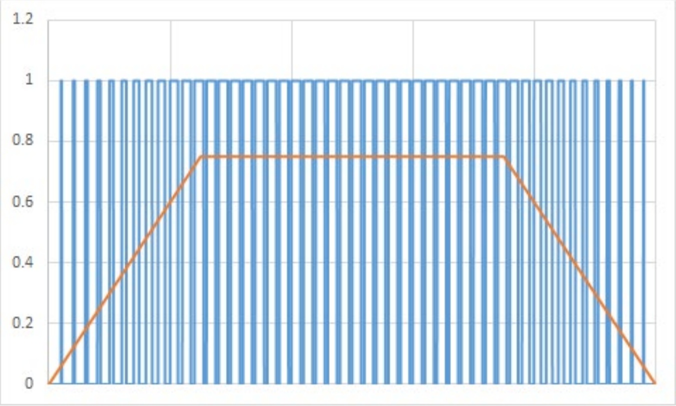


Figure 6 Difference between peak intensity and flash shape peak

In this example, the flash shape peak is 75% of the peak intensity of the light. The reason is that the duty cycle of the switching light is 75% at the flash shape peak. Since the flash intensity is effectively reduced to 75% of its maximum value, this means that the peak-to-effective intensity factor from ANNEX B must also be reduced by the same amount.

For example, in the table for $a = 0.1$ s and non-blue light, the triangular flash with a length of 0.5 s has a peak-to-effective intensity factor of 0.692. If the triangular shape of the flash is created by modulating the light using PWM where the peak of the shape has a duty cycle of 75%, then the peak-to-effective intensity factor is 0.692 x 0.75 = 0.519.

By using the techniques described above, it should be possible to estimate the effective intensity of the most common flashes without needing to use the numerical calculation of Modified Allard Method.

4. CONCLUSIONS

- The Modified Allard Method is the method recommended for determining the effective intensity of a Marine AtoN signal light of any flash profile or multiple flash profiles at any repetition rate.
- The Blondel-Rey method, Equation 1, may be used to determine the effective intensity of *a single flash* of a marine AtoN signal light *providing* the flash profile is *rectangular*. It should not be used for repeating flashes that flash at a rate greater than 60 flashes per minute.

- If, and only if, it is impossible to measure the variation of instantaneous intensity with time, an estimation of effective intensity may be calculated from the Blondel-Rey formula, Equation 1, using values of I_o and t calculated by methods outlined in IALA Recommendation R0205 (E200-5).
- If the approximate flash shape and duration is known, then the peak-to-effective intensity factors given in Annex B can be used to estimate the effective intensity of such flashes.

5. REFERENCES

[1] Mandler and Thacker, “A Method of Calculating the Effective Intensity of Multi-Flick Flashtube Signals”, US Coast Guard Publication CG-D-13-86 (1986)

[2] Ifeachor, E.C., Jervis, B.W.; “Digital Signal Processing – A Practical Approach”, Addison-Wesley

[3] Jackson, L.B.; “Signals, Systems and Transforms”, Addison-Wesley

[4] Smith, S.W.; “The Scientist and Engineer’s Guide to Digital Signal Processing”; Available at http://www.dspguide.com/

ANNEX A COMPUTATIONAL CONSIDERATIONS OF THE MODIFIED ALLARD METHOD

A.1. DIRECT CONVOLUTION

The Modified Allard Method of calculating effective intensity is achieved by mathematical convolution. This process can better be described by considering the discrete data resulting from a measurement of the variation intensity over time with a digital recording device. Figure 7 is a typical flash profile from a rotating beacon and, with it, the visual impulse function.

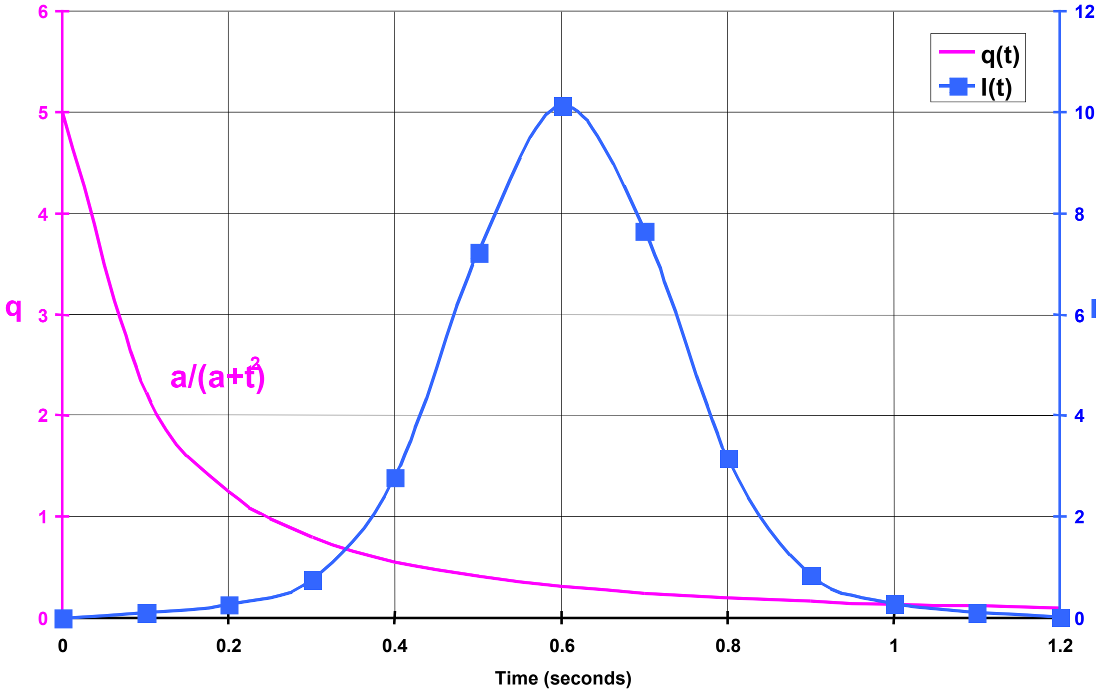


Figure 7 Plot of intensity against time, I(t), and visual impulse function, q(t)

The squares marked on the flash plot are instances in time when the instantaneous intensity was recorded digitally. Both flash profile and visual impulse function can be shown as discrete values by a histogram.

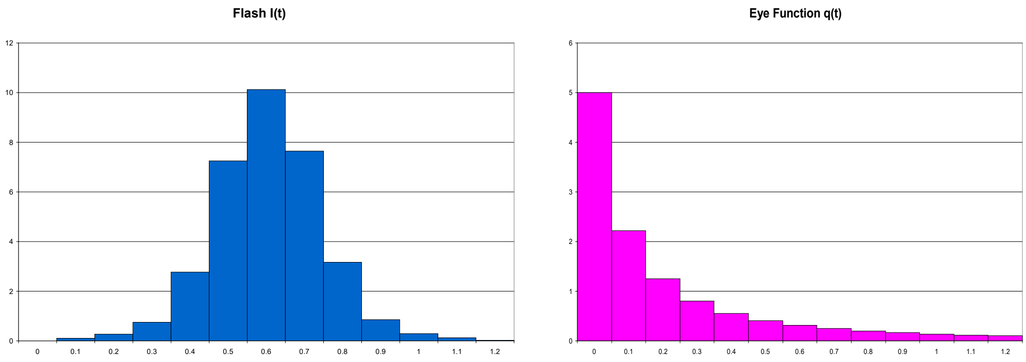


Figure 8 Histograms of flash profile, I(t), and the visual response function, q(t)

The convolution is achieved by stepping the reverse visual impulse function past the flash profile taking the sum product at each step as follows:

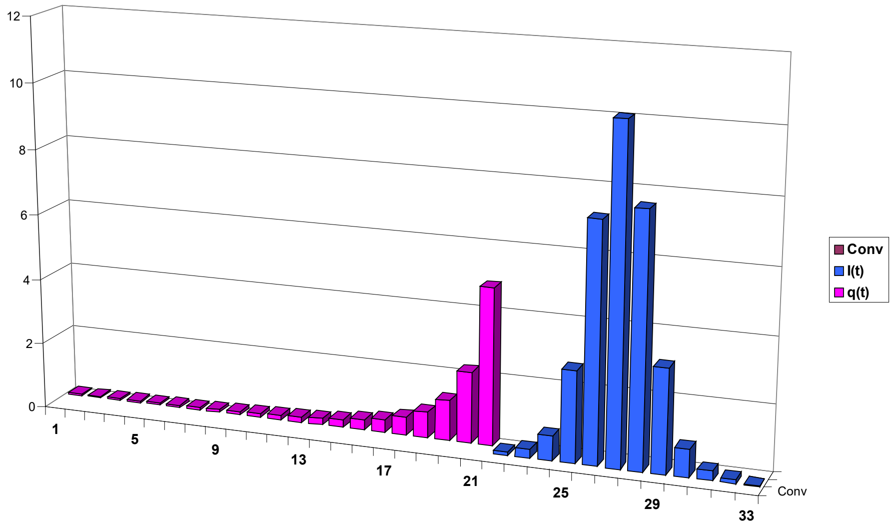


Figure 9 Convolution at t = 0

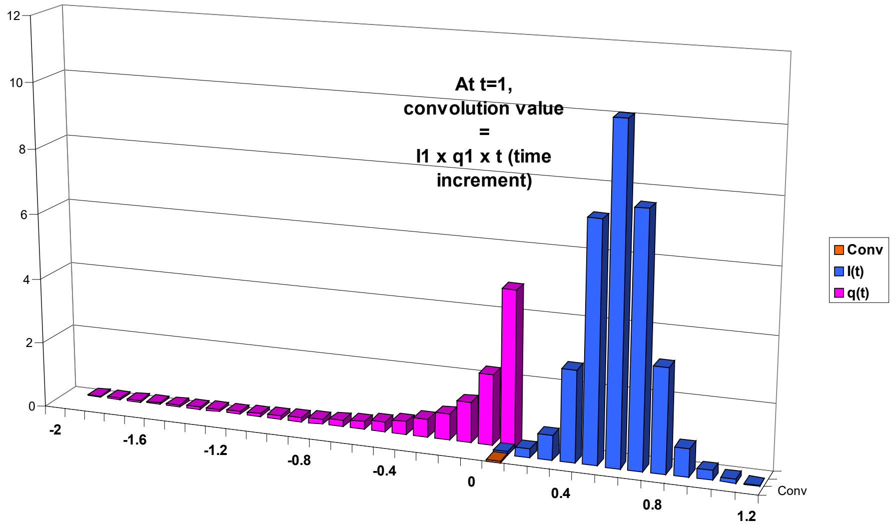


Figure 10 Convolution at t = 1

At the first step, the value of q1 in the visual impulse function is multiplied by the value of I1 in the flash profile. This product is multiplied by the time increment in seconds to give the convolved value for t=1.

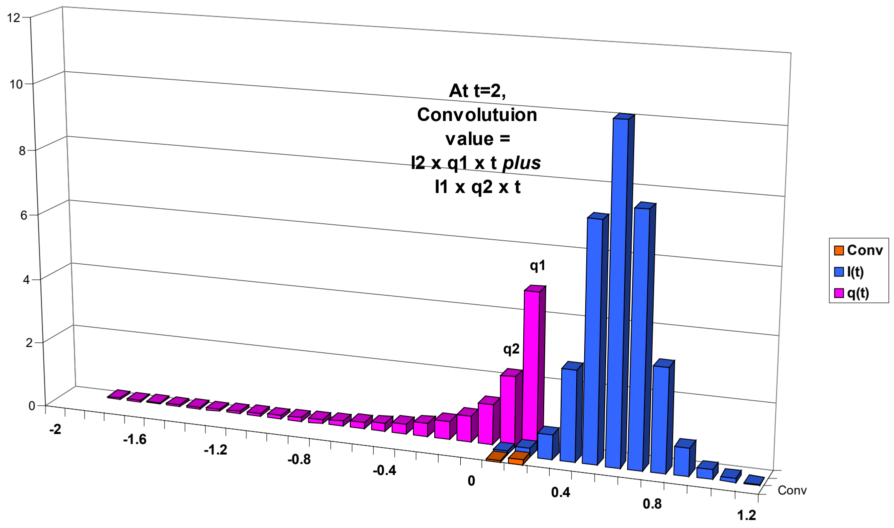


Figure 11 Convolution at t = 2

At t=2, the value of q1 is multiplied by the value of I2, then the value of q2 is multiplied by I1. Both products are then added together and multiplied by the time increment. The result is the convolved value for t=2.

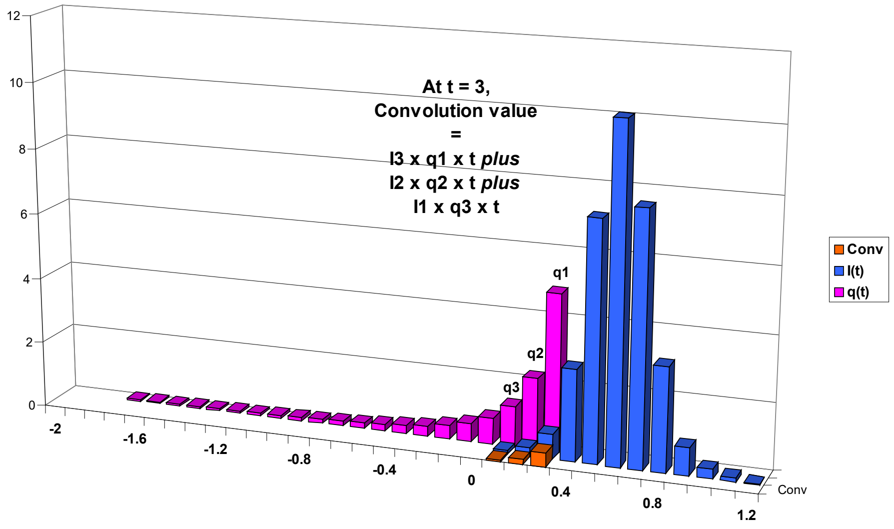


Figure 12 Convolution at t = 3

At t=3, the value of q1 is multiplied by the value of I3, the value of q2 is multiplied by I2 and the value of q3 is multiplied by I1. These three products are then added together and multiplied by the time increment to obtain the resultant convolved value for t=3.

As this process is continued through steps 0 to 9 it is possible to see the convolution plot emerging:

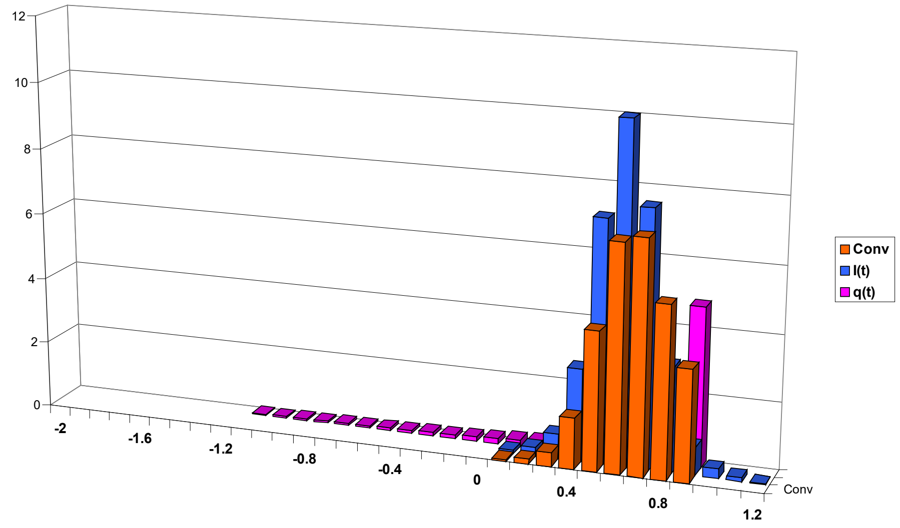


Figure 13   Convolution at t = 9 showing a maximum value at t = 7

Although rather crude, the histograms show the convolution process in discrete format. Reverting to the continuous format, the peak value of the convolution can be taken as the effective intensity value.

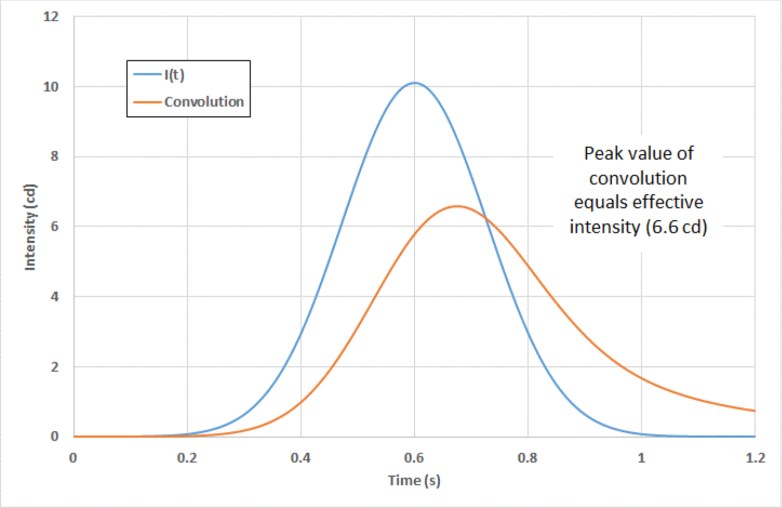


Figure 14   Continuous graph of flash profile I(t) and convolution product

The discrete values of the flash profile, reversed visual impulse function, and time increments can be entered into a spreadsheet. The SUMPRODUCT function may be employed to give a value of the convolution at each time increment. Of the resultant convolved values shown at each time increment, the maximum value should be taken to obtain the effective intensity.

A.2. FREQUENCY-DOMAIN METHOD

The direct convolution method described above is usually sufficient for small datasets. However, it often the case that the datasets can be rather large, and any computation of the Modified Allard Method becomes unwieldy and very slow.

This can be overcome by considering that convolution in the time domain is the equivalent to multiplication in the frequency domain. This is known at the Convolution Theorem and provides for an alternative means of calculating the Modified Allard Method. This theorem is well-known in the field of digital signal processing, especially with regards to digital filtering techniques. More information can be found in [2], [3] and [4]. Using this theorem, we find

$$d * q = \mathcal{F}^{-1}[\mathcal{F}(d) \cdot \mathcal{F}(q)]$$

Equation 6 Convolution Theorem

Where $*$ is the convolution operator

d is the data series

q is the visual system response function

$\mathcal{F}$ is the Fourier transform function

$\mathcal{F}^{-1}$ is the inverse Fourier transform function

As can be seen from Equation 6, it is necessary to convert the time-domain data into the frequency-domain, carry out the multiplication, and then convert the result back into the time-domain. On the face of it, this may seem more arduous than the direct convolution method described above. However, this technique is capable of handling large sets of data much better than the direct convolution method.

In field of digital signal processing, the Fast Fourier Transform (FFT) is a popular technique to translate discrete time-domain data into the frequency-domain. This Guideline will not consider how to implement a FFT routine, since significant information and many software libraries are available on-line to aid implementation (For example, here: https://rosettacode.org/wiki/Fast_Fourier_transform).

The method is described below and uses the FFT, and its inverse, to implement the Modified Allard Method.

The steps to be taken are as follows:

1 Create and populate an array of values of $q(t)$ that is the same length as that of the signal array;

2 Expand the signal and $q(t)$ arrays so that their lengths are equal to the next highest value of 2^N, where N is an integer. Fill the extra space with zeros;

3 Double the length of the signal and $q(t)$ arrays, again filling the extra space with zeros;

4 The values of the signal and $q(t)$ arrays will form the real parts of the complex numbers supplied to the FFT routine. The imaginary parts are all set to zero;

5 Transform both the signal and $q(t)$ complex numbers using FFT;

6 Perform complex number multiplication on the results of the two FFT transformations;

7 Transform the complex number multiplication result using inverse FFT;

8 Using only the real part of the inverse FFT result, multiply the values by the sampling period. The imaginary part can be discarded;

9 The result is an array equal to the result of MAM. The maximum value of the array is the effective intensity.

A 1.1. PERFORMANCE

Despite the more complex method of preparing the data, transformations and multiplications, the technique has proved to be much faster than the convolution method with equal accuracy. As an example, using a 5,000-element dataset in a spreadsheet, an optimised MAM convolution macro took approximately 3.1 s to calculate the effective intensity. By applying the technique described above to the same data using a macro, the calculation took just 0.23 s. For an 80,000-element dataset, the above technique took just 4.5 s.

ANNEX B PEAK-TO-EFFECTIVE INTENSITY FACTORS FOR COMMON FLASH SHAPES

This annex provides the peak-to-effective intensity (I_e/I_o), factor for a number of common flash shapes calculating using the Modified Allard Method. Two sets of tables are provided to account for the different visual constant in use depending on the nature and observation conditions. In all cases, the flash length, T, is the duration of the flash at 50% of peak intensity.

The simplified flash shapes have been chosen so that they represent most of the light flashes currently in use. They are:

- Rectangular – These flashes are generally created using LEDs in fixed optics.
- Trapezoid – This shape can be created by a rotating shade inside a fixed optic. Some high-powered LED lights inside fixed optics have a similar shape to limit inrush currents.
- Single-sided Trapezoid/Triangular – These are more specialist type of flash created using LEDs.
- Incandescent – These flashes are created by using incandescent lamps in fixed optics.
- Cosine-Squared/Gaussian – These flashes can be created by rotating optics.

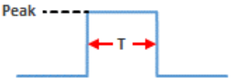
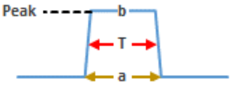
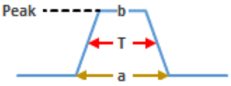
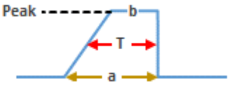
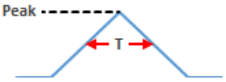
More complex flash shapes or measured flash profiles should be calculated using the Modified Allard Method directly.

These tables are used to estimate the effective intensity of a flash given its flash shape and duration without needing to carry out the Modified Allard Method calculation. These factors can be used directly for the de-rating factor k_e in the flash profile calculations described in IALA Guideline *G1148*. For example, if we have a white 0.5 s triangular flash with a peak intensity of 1500 cd, then we find the effective intensity factor from the table in Section B1 below to be 0.692. Therefore, the effective intensity for this flash is 1500 x 0.692 = 1038 cd.

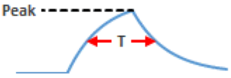
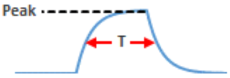
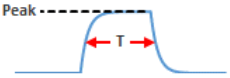
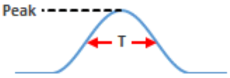
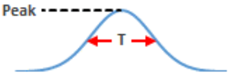
Allowance must be made for pulse-width modulated light if the peak intensity of the flash shape is less than that when the light when fully on. In this case, the values in the tables must be multiplied by the duty cycle at the flash shape peak, e.g. a factor of 0.75 should be applied if the duty cycle at flash shape peak is 75%.

B.1. FOR ALL SIGNAL LIGHTS EXCEPT BLUE AT NIGHT

Peak-to-Effective Intensity Factor for All Signal Lights except Blue Lights at Night (a = 0.1 s)

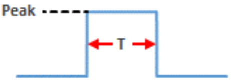
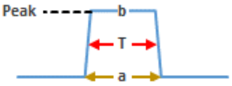
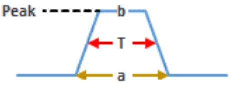
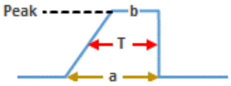
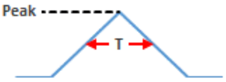
Flash Length to 50% of peak, T (seconds)	Rectangular 	Trapezoid (b = 5a/6) 	Trapezoid (b = a/2) 	Trapezoid (b = a/2) 	Triangular 
0.01	0.091	0.091	0.089	0.092	0.084
0.02	0.167	0.166	0.160	0.165	0.147
0.05	0.333	0.330	0.313	0.330	0.274
0.1	0.500	0.494	0.467	0.496	0.398
0.2	0.667	0.660	0.630	0.661	0.530
0.5	0.833	0.829	0.806	0.829	0.692
1	0.909	0.906	0.892	0.906	0.791
2	0.952	0.951	0.943	0.943	0.865
5	0.980	0.980	0.976	0.972	0.929

Peak-to-Effective Intensity Factor for All Signal Lights except Blue Lights at Night (a = 0.1 s)

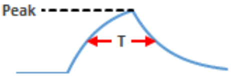
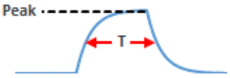
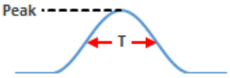
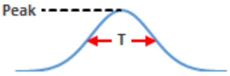
Flash Length to 50% of peak, T (seconds)	Incandescent (τ = T/2) 	Incandescent (τ = T/5) 	Incandescent (τ = T/10) 	Cosine-Squared 	Gaussian 
0.01	0.085	0.086	0.088	0.086	0.088
0.02	0.147	0.153	0.160	0.152	0.155
0.05	0.274	0.300	0.316	0.290	0.293
0.1	0.403	0.451	0.476	0.426	0.429
0.2	0.547	0.613	0.642	0.571	0.573
0.5	0.723	0.792	0.816	0.741	0.741
1	0.825	0.881	0.899	0.837	0.836
2	0.895	0.936	0.946	0.903	0.902
5	0.950	0.973	0.978	0.954	0.954

B.2. FOR BLUE LIGHTS AT NIGHT ONLY

Peak-to-Effective Intensity Factor for Blue Signal Lights at Night Only (a = 0.2 s)

Flash Length to 50% of peak, T (seconds)	Rectangular 	Trapezoid (b = 5a/6) 	Trapezoid (b = a/2) 	Trapezoid (b = a/2) 	Triangular 
0.01	0.048	0.048	0.047	0.048	0.046
0.02	0.091	0.091	0.089	0.090	0.084
0.05	0.200	0.199	0.191	0.198	0.173
0.1	0.333	0.330	0.313	0.331	0.274
0.2	0.500	0.494	0.467	0.495	0.398
0.5	0.714	0.708	0.678	0.709	0.572
1	0.833	0.829	0.806	0.829	0.692
2	0.909	0.906	0.892	0.892	0.791
5	0.962	0.960	0.954	0.946	0.884

Peak-to-Effective Intensity Factor for Blue Signal Lights at Night Only (a = 0.2 s)

Flash Length to 50% of peak, T (seconds)	Incandescent (τ = T/2) 	Incandescent (τ = T/5) 	Incandescent (τ = T/10) 	Cosine-Squared 	Gaussian 
0.01	0.047	0.046	0.047	0.046	0.048
0.02	0.085	0.085	0.088	0.086	0.088
0.05	0.173	0.182	0.191	0.180	0.184
0.1	0.274	0.300	0.316	0.290	0.293
0.2	0.403	0.451	0.476	0.426	0.429
0.5	0.593	0.662	0.691	0.616	0.617
1	0.723	0.792	0.816	0.741	0.741
2	0.825	0.881	0.899	0.837	0.836
5	0.912	0.948	0.957	0.919	0.918